Chapter 17 Interaction of Radiation with Matter

17.1 Introduction

At this point we have described nuclear transitions and reactions that produce various forms of nuclear radiation. The radiation propagates out from the originating nucleus and interacts with other matter along its path. These interactions with external matter allow us to observe the radiation, and its effects, and to determine the nature of the transition inside the nucleus. The interaction of radiation with matter is also the cause of chemical, physical and biological changes that concern the public at large. We will specifically address the operating principles of radiation detectors in the next chapter but first we will consider the fundamental interactions of nuclear radiation with matter.

It should be clear that radiation (with the exception of neutrons) primarily interacts with bound electrons. For example, a silicon atom contains fourteen electrons in a sphere with a radius of 0.12 nm that presents a geometrical cross section of $4.5 \times 10^{-20} \text{m}^2$. The nucleus at the center of the sphere has a radius of 3.6 fm with a geometrical cross section of $4.1 \times 10^{-29} \text{m}^2$. The geometrical probability to strike the electrons in an atom is something like nine orders of magnitude higher than that to strike the nucleus. A scattered electron leaves the original atom and creates an ion-pair. The interaction of a single particle of nuclear radiation can lead to tens or hundreds of thousands of ion-pairs and so nuclear radiation is generally called ionizing radiation.

From the starting point that we should consider the interaction of the radiation with electrons, we can divide nuclear radiation into four classes as
indicated in Table 17-1. The overall scattering of the particle from an electron will be dominated by the mass and the charge on the particle. Thus, massive charged particles will tend to scatter the small mass electrons widely without losing much energy, while the collision of an electron with another electron will lead to energy sharing, and a photon can be scattered or even absorbed by a single electron. Neutrons only interact very weakly with electrons through their small magnetic moment and predominately interact with nuclei. (As will be discussed later, neutrons are very penetrating and difficult to detect due to the small probability of striking a nucleus.) As a general rule, all of the interactions end up in creating energetic electrons. The heavy charged particles and recoiling atoms scatter electrons. The electrons interact with matter to create moving charged particles, while neutrons create recoiling nuclei and photons create moving electrons.

We will discuss the interaction of each class of radiation starting with the simplest, those charged particles that are more massive than electrons. We will consider what happens as they pass through various types of matter. The radiation generally penetrates through many, many atomic layers so we can generally assume that the atoms are randomly distributed in space. This assumption is certainly true for liquids and gases but we usually think of solids as having a regular crystal or lattice structure. The solid material that we encounter in everyday life certainly has a lattice structure on the microscopic scale but nearly all materials are polycrystalline on a larger scale. Thus, the types of radiation that we will generally consider will cross many crystal boundaries in normal materials. The exceptions are single crystals of silicon, germanium, or other special materials that are used in
semiconductor-based radiation detectors that rely on their special electronic properties. We have to be aware of the orientation of the crystal axes in these devices relative to the propagation direction of the radiation.

Also before starting the discussion we should define a quantity called the areal density. We have said that ionizing radiation will pass through a significant amount of material and sometimes will even pass completely through an object. We can imagine that the solid piece of material can be compressed into a thin, two-dimensional sheet as far as the radiation is concerned. The origins of this concept lie in early experiments with alpha rays in which very thin sheets of metals were necessary. We can easily measure the length, $x$, and breadth, $y$, of such thin foils but it is very difficult to measure the thickness, $z$, without destroying the foil. On the other hand, we can determine the mass, $m$, of the foil rather easily and then calculate the thickness using the density of the material. For example, if the foil is made from a pure chemical element, so that the density is known, then:

$$\text{linear thickness } = z = \frac{m}{x \times y} = \frac{\text{areal density}}{\rho}$$

where $\rho$ is the density (and has dimensions of mass per unit volume, of course).

The measured quantity of mass per unit area is often used to characterize thin foils of pure materials. This quantity is called the areal density. Typical dimensions for materials that we might use in experiments are mg/cm$^2$. Thus, if the density of a typical metal were in the range of 5-10 g/cm$^3$, the thickness of a typical foil would be on the order of fractions of a millimeter.
Example of Areal Density

A ream of a certain type of standard letter-sized paper (in the United States) was found to have a mass of 2.26 kg. What is the areal density of one sheet of this paper? Recall that a ream of paper contains 500 sheets, so that the areal density of one sheet is simply:

\[ A = \frac{\text{total mass}}{\text{surface area}} \]

\[ A = \frac{2260 \text{ g}}{(500 \times 8.5 \text{ in} \times 11 \text{ in}) \times (2.54 \text{ cm/in})^2} \times 1000 \text{ mg/g} = 7.5 \text{ mg/cm}^2 \]

17.2 Heavy Charged Particles (A ≥ 1)

We can imagine the progress of any ionizing radiation through material as a series of straight-line segments between scattering events. The scattering events primarily involve electrons. The total path is thus made up of these line segments and the overall trajectory of the particle in the material will depend on the kinematics of these scattering events. Elastic scattering is, of course, governed by the conservation of momentum and energy so we should expect that the mass of the particle will play a large role in determining the overall features of the trajectory of the particle.

The tracks of a few heavy charged particles that have stopped in a photographic film are shown in Figure 17-1. The photographic film, called a nuclear emulsion, is sensitive to the ionization that is caused by the charged particles as they move. Normally visible photons “expose” photographic film by creating
photoelectrons, and the ionization is converted into an image through the
development process. All of the charged ions that we have to consider have positive
charges. As can be seen in figure 17-1, energetic charged ions move through
material on essentially straight trajectories, giving up or losing kinetic energy
through collisions with the atomic electrons of the material. Only rarely by
comparison is an ion scattered by the Coulomb potential of a nucleus, and even
more rarely does a nuclear reaction take place. Nuclear reactions are excluded when
the initial kinetic energy of the heavy charged particle is lower than the Coulomb
barrier (as discussed in Chapter 10). Thus the ions interact with an extremely large
number of electrons and we can examine the average behavior of the ions as they
pass through material.

The rate at which charged particles lose energy at they travel through a given
material is called the stopping power of that material. The stopping power is made
up from two parts, the electronic stopping power due to the interaction with the
atomic electrons of the material and the nuclear stopping power. Thus:

\[ -\frac{dE}{dx} = S_{\text{electronic}} + S_{\text{nuclear}} = S_{\text{electronic}} \]

17-1

because the electronic stopping power is always much larger than the nuclear
stopping power. Notice that the minus sign on the rate to indicate that the ions are
losing kinetic energy. The nuclear stopping power is not zero, of course, because we
know that nuclear reactions do take place even if they are rare. The stopping powers
are functions of the mass, charge and velocity of the ion, the atomic number and density of the medium.

Niels Bohr suggested that the energy loss rate could be estimated in a very simple picture as the series of impulses delivered to individual electrons by the ion. Imagine an ion moving on a straight-line trajectory past an electron. (see Figure 17-2) A net impulse to the electron will occur in the direction perpendicular to the trajectory of the ion because any impulse due to the approaching ion will be cancelled by that of the receding ion. It can be shown that the energy gained by a single electron and thus lost by an ion in one encounter depends on the impact parameter, b, as:

\[ \Delta E(b) = \frac{2q^2e^4}{m_e v^2 b^2} \]

where \( q \) is the charge of the ion (often equal to the atomic number), \( v \) is the velocity of the ion, and \( m_e \) is the mass of the electron. This expression can be converted to a differential expression by multiplying by the electron number density, \( N_e \), times the volume element:

\[-dE(b) = \Delta E(b)N_e dV = \Delta E(b)N_e 2\pi db dx \]

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by using the cylindrical coordinates of impact parameter and taking \( x \) along the ion's path. This expression should not be integrated from \( b=0 \) to \( b=\infty \) but only over the range \( b_{\text{min}} \) to \( b_{\text{max}} \) that are appropriate to the initial assumptions with regard to the ion and the electron, so that:
The minimum impact parameter will correspond to those collisions in which the maximum amount of kinetic energy is transferred to the electron. Due to conservation of momentum, the maximum electron energy is $W_{\text{max}} = (1/2)m_e(2\gamma v)^2$ where we have included the relativistic factor $\gamma$ due to the low mass of the electron. (Recall that $\gamma = \sqrt{\frac{1}{1 - \beta^2}}$ and $\beta = v/c$.) Thus, substituting into the expression for the energy loss at a given impact parameter:

$$\Delta E(b_{\text{min}}) = \frac{2q^2\epsilon^4}{m_e\gamma^2b_{\text{min}}^2} = 2\gamma^2m_e\nu^2$$

we find that:

$$b_{\text{min}} = \frac{q\epsilon^2}{\gamma m_e\nu^2}$$

The maximum impact parameter has to be estimated from different considerations. The basis of this process is that the ion rapidly moves past the electron and delivers a sharp impulse to the electron. The electrons are bound in atoms and thus are orbiting with their own characteristic frequencies or time-scales. Thus, the time for the ion to cross the atom should be less than the average time for an electron orbit; otherwise the collision will not be adiabatic or "rapid." The time for
the ion to move past can be estimated as the ratio of the impact parameter to the
ion’s velocity, the average orbital time for an electron will clearly depend on the
chemical element, as there will be an average radius and velocity, thus:

\[ \frac{b_{\text{max}}}{\gamma v} = \frac{R_e}{v_e} = f(Z) \]

where \( f(Z) \) is a function of the atomic number of the stopping material. So that we
can combine these two limits into the expression for the stopping power or energy
loss rate to get Bohr’s classical formula:

\[ \left( -\frac{dE}{dx} \right)_{\text{Bohr}} = 4\pi q^2 e^4 N_e \gamma^2 m_e v_e^3 f(Z) \left( \frac{N_A}{N_e} \right) \]

This expression has been superseded by the expression derived by Bethe and Bloch
based on momentum transfer in a quantum mechanically correct formalism. Their
expression with the expanded form of the electron number density is:

\[ \left( -\frac{dE}{dx} \right)_{\text{Bethe–Bloch}} = 4\pi N_A r_e^2 m_e c^2 \rho \frac{Z q^2}{A \beta^2} \left[ \ln \left( \frac{W_{\text{max}}}{I} \right) - \beta^2 \right] \]

where \( N_A \) is Avogadro’s number, \( r_e \) is the classical radius of the electron, \( \rho \) is the
density of the stopping medium with atomic number, \( Z \), mass number, \( A \), and
ionization potential, \( I \). Finally, \( W_{\text{max}} \) is the maximum energy transfer, encountered
above. The structure is very similar to the classical formula as should be expected
but it includes an extra term in the logarithm. Various formulas are available to give the average variation of the ionization potential for the chemical elements. For example, the expressions:

\[
\frac{I}{Z} = (12 + 7Z^{-1}) \text{ eV}, \quad Z < 13
\]

\[
\frac{I}{Z} = (9.76 + 58.8Z^{-1.19}) \text{ eV}, \quad Z \geq 13
\]

Is the result from one empirical fitting of the data but one should realize that the variation could be quite complicated due to the filling of the atomic shells.

**Example of the use of the Bethe-Bloch formula**

Evaluate the stopping power of beryllium metal for \(^{18}\text{O}^+\) ions with a kinetic energy of 540 MeV (\(E/A=30\) MeV) using the Bethe-Bloch formula. Finding some necessary constants, the density of beryllium metal is 1.85 g/cm\(^3\) and \(Z=4\) so that the ionization potential can be estimated as:

\[
\frac{I}{Z} = (12 + 7Z^{-1}) \text{ eV} = 13.75 \text{ eV}
\]

\(I=55 \text{ eV}\)

\(l=55 \text{ eV}\)

The values of \(\beta\) and \(\gamma\) for the ion can be obtained from the relativistic expressions (derived elsewhere):

\[
\cdots = (1-(m_0^2/(m_0^2+c^2)+(E/A))^{1/2}
\]

\[
\cdots = (1-(931.5/(931.5 + 30))^{1/2}=0.1766
\]

\[
\cdots = (1/(1 - \beta^2)^{1/2} = 1.01598
\]

The value of \(W_{\text{max}}\) is can be evaluated as:

\[
W_{\text{max}} = 2m_e c^2 (\gamma \beta)^2
\]

\[
W_{\text{max}} = 2 \times 0.511 \text{ MeV} \times (1.01598 \times 0.1766)^2 = 0.03291 \text{ MeV}
\]

Finally, the expression with the constants evaluated is:

\[
\left(-\frac{dE}{dx}\right)_{\text{Bethe-Bloch}} = 0.3071 \frac{\text{MeV} \cdot \text{cm}^2}{\text{g}} \rho \frac{Zq^2}{A\beta^2} \left[\ln\left(\frac{W_{\text{max}}}{I}\right) - \beta^2\right]
\]
\[
\left( \frac{-dE}{dx} \right)_{\text{Bethe–Bloch}} = 0.3071 \frac{MeV \cdot cm^2}{g} (1.85 g/cm^3) \frac{4 \times 8^2}{9(0.1766)^2} \left[ \ln \left( \frac{0.03291 MeV}{55 \times 10^{-6} MeV} \right) - 0.1766^2 \right]
\]

\[
\left( \frac{-dE}{dx} \right)_{\text{Bethe–Bloch}} = 518.0 MeV/cm \times \left[ \ln(598.4) - 0.03119 \right] = 3.3 GeV/cm
\]

Notice that the answer indicates that an ion with only 540 MeV of kinetic energy will lose all its energy and stop before it travels a fraction of a centimeter. As discussed later, we need to integrate this expression to determine the predicted range.

The modern form of the stopping power includes two corrections. The first correction applies at high energies at which polarization of electrons by the electric field of the moving ion tends to shield distant electrons; this correction depends on the electron density; it is subtractive and given the symbol $\varepsilon$. The second correction applies at low energies when the collisions are no longer adiabatic, similar to the limit applied by Bohr. This correction is termed the shell correction as it depends on the orbital velocities of the electrons. It is also a subtractive term and given the symbol $C$. If we evaluate all the constants, then the modern form is:

\[
\left( \frac{-dE}{dx} \right)_{\text{Bethe–Bloch}} = 0.3071 \frac{MeV \cdot cm^2}{g} \rho \frac{Zq^2}{A \beta^2} \frac{\ln(W_{\text{max}}/I)}{1} - \beta^2 - \delta - C
\]

which has the dimensions of MeV/cm when the usual form of the density in g/cm$^3$ is used. The actual evaluation of this function is complicated due to the detailed variation of the ionization potential and the two correction terms. The reader is
referred to more detailed discussions [see the book by Leo] for actual formulas for
the correction factors. In practice, several computer codes and detailed tables of the
stopping powers are available. In addition, some authors divide through by the
density, \( \xi \), and report the mass stopping power:

\[
-\frac{1}{\rho} \frac{dE}{dx}
\]

with dimensions of MeV-cm\(^2\)/g which is convenient for combining materials.

If we look at the form of these equations for the stopping power, we would
see that they all have a part that depends on the moving ion and another part that
depends on the stopping medium. If we concentrate on the part that depends on the
ion we find that:

\[
-\frac{dE}{dx} \propto \frac{q^2}{v^2} \ln \left( \frac{v^2}{v^2} \right) g(Z)
\]

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in which we can convert the factors of \( v^2 \) into kinetic energy, \( E \), by suitably applying
factors of \( 1/2 \) \( m_{\text{ion}} \). The function \( g(Z) \) collects all the variation on the absorbing
medium. The revised expression shows that the energy loss rate will be
proportional to the mass of the ion:

\[
-\frac{dE}{dx} \propto \frac{Aq^2}{2E} \ln \left( \frac{\gamma^2 2E}{A} \right) g(Z)
\]

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and inversely proportional to the kinetic energy. At low ion velocities (\( E/A < 10 \)
MeV/A), the \( \ln \left( \frac{\gamma^2 2E}{A} \right) \) term is approximately constant and
Thus, a more energetic ion will tend to lose energy at a lower rate than a less energetic ion. Be careful to note that we have ignored the relativistic terms, $\gamma^2$ and $\beta^2$, in the parentheses which produce a minimum in the complete function near $\beta \sim 0.96$ and a small rise at higher velocities. (Particles with $\beta \sim 0.96$ are called *minimum ionizing particles.*) The proportionality of the stopping power on the mass and square of the charge of the ion for a given kinetic energy provides the basis for a very effective particle identification using thin silicon semiconductor detectors, as discussed in Chapter 18.

We are now in a position to examine the slowing down of a charged particle as it penetrates into material. Kinetic energy is lost through scattering electrons away from the essentially straight-line path of the ion. If the initial kinetic energy of the ion is a few MeV/A or higher, the rate at which kinetic energy is dissipated slowly increases as the ion penetrates into the material. For example, the stopping power of beryllium metal for a very energetic $^{40}$Ar ion is shown in figure 17-3 along with the residual energy of the ion. (The thickness scale can be converted into a linear distance by dividing by the density in appropriate units.) Notice that the stopping power is relatively constant over most of the ion’s path. The kinetic energy of the ion uniformly decreases as it moves through the material. However, two changes occur as the velocity of the ion approaches the Bohr velocity of the atomic electrons, $v_{\text{Bohr}} = \frac{Z \hbar}{m_e n a_0} = 5.51 \times 10^{-3} \frac{Z_c}{n}$. The energy loss rate begins to increase dramatically
as $\beta \to 0$ but more importantly, the charge state on the ion starts to decrease as the ion captures orbital electrons causing the rate to drop. As indicated in figure 17-3, the ion rapidly loses energy at the end of its range and stops rather suddenly. The energy loss for an alpha particle near the end of its range is shown in figure 17-4. The resulting peak in the energy loss function just before the end of the charged particle's range is called the *Bragg Peak*. The fact that charged particles deliver a significant fraction of their kinetic energy at the end of their range makes charged particles useful for radiation therapy.

All of these expressions for the stopping power only apply to pure chemical elements. The stopping power of a compound or any complicated mixture will depend on the overall density and the relative numbers of electrons from each chemical element. Recognizing that the ionizing radiation will usually move through macroscopic distances we can use an averaging procedure called *Bragg’s Rule*. The average mass stopping power is:

$$
\left( \frac{1}{\rho} \frac{dE}{dx} \right)_{\text{total}} = \frac{w_1}{\rho_1} \left( \frac{dE}{dx} \right)_1 + \frac{w_2}{\rho_2} \left( \frac{dE}{dx} \right)_2 + \frac{w_3}{\rho_3} \left( \frac{dE}{dx} \right)_3 + \cdots
$$

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where $w_1$, $\rho_1$, etc. refer to the fraction by mass of element 1 in the entire mixture, and its’ elemental density. The sum ranges over all the elements in the mixture. Thus, if the mixture was a pure compound then we would combine the numbers of each element in the molecular formula. If the mixture had several components then we would combine the masses of each element from all the components and so on to get an overall mass stopping power.

**Example**
What is the rate of energy loss of an 8 MeV α-particle in air? Assume air is 21% oxygen and 79% nitrogen.

\[
\left(\frac{1}{\rho} \frac{dE}{dx}\right)_{\text{total}} = \sum_i w_i \frac{1}{\rho_i} \left(\frac{dE}{dx}\right)_i
\]

where \( \rho \) is the total density, and \( \rho_i \) the density of the \( i \)th element.

For oxygen

\[
\left(\frac{1}{\rho} \frac{dE}{dx}\right)_{\text{oxygen}} = 0.3070 \frac{\text{MeV} \cdot \text{cm}^2}{\text{g}} \frac{Zq^2}{\ln 0.002194} \left[ \ln \left(\frac{w_{\text{max}}}{I}\right) - \beta^2 \right]
\]

neglecting any correction terms

\[
\frac{I}{Z} = (12 + 7Z^{-1}) \text{eV} = 12.875
\]

\[
I = 103 \text{eV}
\]

\[
\beta = \left[ 1 - \frac{m_0c^2}{m_0c^2 + E/A} \right]^{1/2} = 0.04629
\]

\[
\gamma = \left[ \frac{1}{(1 - \beta^2)} \right]^{1/2} = 1.0011
\]

\[
w_{\text{max}} = 2 \times 0.511 (\gamma \beta)^2 = 0.002194 \text{MeV}
\]

\[
\left(\frac{1}{\rho} \frac{dE}{dx}\right)_{\text{oxygen}} = 0.3070 \frac{8x^2}{16(0.04629)^2} \left[ \ln \left(\frac{0.002194}{103 \times 10^{-6}}\right) - (0.04629)^2 \right]
\]

For nitrogen

\[
\frac{I}{Z} = (12 + 7Z^{-1}) \text{eV} = 13
\]

\[
I = 91 \text{eV}
\]

\[
\left(\frac{-1}{\rho} \frac{dE}{dx}\right)_{\text{nitrogen}} = 0.3070 \frac{7x^2}{14(0.04629)^2} \left[ \ln \left(\frac{0.002194}{91 \times 10^{-6}}\right) - (0.04629)^2 \right]
\]

\[
= 911.4 \text{MeV/} \text{g/cm}^2
\]
For air

\[
\left(-\frac{1}{\rho} \frac{dE}{dx}\right)_{\text{air}} = 0.21\left(-\frac{1}{\rho} \frac{dE}{dx}\right)_{\text{oxygen}} + 0.79\left(-\frac{1}{\rho} \frac{dE}{dx}\right)_{\text{nitrogen}}
\]

\[= 0.21(875.1) + 0.79(911.4) = 903.8\,\text{MeV/g/cm}^2\]

One of the implications of the stopping power formulas for heavy charged particles is that all particles of a given type will follow the same energy loss pattern in a given material. More specifically, the example shown in figures 17-3 started with 8 GeV $^{40}$Ar ions in Be. However, these curves display the expected result for all $^{40}$Ar ions with kinetic energies less than 8 GeV. The energy loss rate for an ion with 4 GeV or even 4 MeV can be read off the graph by finding when the residual energy of the ion is equal to the required energy. This may seem a trivial point but it has the more subtle meaning that all the ions will follow exactly the same energy loss pattern, within the limits of the statistical process, if we ignore nuclear Coulomb scattering. Formally we can write that the amount of kinetic energy lost, $\Delta E$, in a finite thickness, $\Delta x$, of material is:

\[
\Delta E(\Delta x) = \left(\frac{dE}{dx}\right)\Delta x
\]

and the statistical variation in the energy lost $\delta\Delta E$ would be evidenced as a width in the measured value that is called the amount of energy straggling. In a colloquial expression, the ions are said to straggle through the material and the width of the energy distribution is due to this straggling. Thus, as ions pass through matter the
spread in their energies increases (Fig 17-5). This can be represented mathematically as a Gaussian distribution

\[
\frac{N(E)dE}{N} = \frac{1}{\alpha \pi^{1/2}} \exp \left[ -\frac{(E - \bar{E})^2}{\alpha^2} \right]
\]

17-17

where the straggling parameter \( \alpha \), which is the half-width at (1/e)th height, is given by the expression

\[
\alpha^2 = 4\pi q^2 e^4 N_e x_0 \left[ 1 + \frac{kI}{m_v v^2} \ln \left( \frac{2m_v v^2}{I} \right) \right]
\]

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where \( k \) is a constant (about 4/3) and the thickness of absorber the ion has penetrated is \( \rho x_0 \).

The range or distance that a heavy charged particle will travel in a material can be obtained by integrating the energy loss rate along the path, of the ion. In the approximation that the ion follows a straight line trajectory then the range for a given kinetic energy, \( R(T) \), would be given by the integral:

\[
R(T) = \int_0^r \left( \frac{dE}{dx} \right)^{-1} dE
\]

17-19

where the function \( dE/dx \) is the appropriate function for the ion in the material. There are two difficulties in applying this simple integral, the ions will suffer a different number of collisions with atomic electrons and, more importantly, the ions will undergo some scattering from the Coulomb fields of the atomic nuclei. The
multiple Coulomb scattering leads to an effect that the ion's trajectory is not straight but rather is made up from a series of straight line-segments. Thus, the apparent range or the projection of the range onto the initial velocity vector of the ion will not be a single value but rather will consist of a statistical distribution of values. Thus, the distribution of ranges is due to range straggling. It is important to note that size of range straggling will grow as an ion penetrates into material because it will literally add-up, in contrast to the energy straggling mentioned above. The range of an ion and its fluctuations are integral quantities whereas the energy loss rate and its fluctuations are differential quantities. It is still true that Coulomb and nuclear scattering are relatively rare so that the range straggling for typical ion-energies in metals is on the order of a few percent of the range. The qualitative features of the range distribution and the attenuation curve for a typical heavy charged particle are shown in figure 17-6. Heavy charged particles penetrate uniformly into matter with essentially no attenuation in intensity until they are nearly at rest, at this point the intensity of moving ions rapidly drops to zero.

The calculated range-energy curves for some low-mass charged particles in silicon are shown in figure 17-7. We can see from the integral form of the range as a function of initial kinetic energy, given above, that $R \propto aE^b$. The exponent should be of order two at low energies where the energy loss rate is dominated by the $1/\beta^2$ or $1/E$ term. The range-energy relationships are very useful in determining the kinetic energies of particles by measuring the attenuation curves. More recently, range-energy relationships are used to identify charged particles that are detected in silicon semiconductor telescopes as they emerge from nuclear reactions. The
scaling rules that apply to the stopping power for different ions in a given medium can be extended to the range-energy relationship. For example, given the range of ion “1” at an initial kinetic energy $T_1$, the range of ion “2” with a different mass, charge, and kinetic energy is:

$$R_2(T_2) = \frac{M_2}{M_1} \frac{q_1^2}{q_2^2} R_1(T_2 \frac{M_1}{M_2})$$

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Notice that we have to scale the range of the known, first particle at the appropriate energy of the unknown, second particle.

As a final, more practical point about the stopping power and ranges of charged particles we should consider the best method to calculate the amount of energy deposited in a thin foil. Clearly the ion will slow down as it passes through the material so that the energy loss rate will change as the particle passes through the foil. Thus, we should use the average energy loss rate but notice that the function is not linear so that we will need a technique to determine the average. Two cases can be identified: thin foils in which the initial, average, and final energy loss rates are nearly the same, and thick foils in which the particle undergoes a substantial energy loss. In the former example of a thin foil, we can use the expression written above that:

$$\Delta E(\Delta x) = \left( \frac{dE}{dx} \right) \Delta x$$

and we should verify that the final rate is approximately equal to the initial rate:

$$\left( \frac{dE}{dx} \right)_{\text{initial}} = \left( \frac{dE}{dx} \right)_{\text{final}}$$
If the energy loss rates are not substantially different, then we can use the initial rate to obtain the average in a successive approximation procedure. For the case of substantial slowing, we can recall that the range relations come from the integration of dE/dx and thus provide the average energy loss rate that we should use. The technique relies on determining the ranges of ions in graphs or tables of ranges as follows: Imagine that an incident particle with an energy, \( E_0 \), passes through some material with thickness, \( t \). These are the “known quantities.” The particle will emerge from the foil with an energy, \( E_1 \), which we would like to determine. We can find the total range of the ion in the material from tables, \( R_0 \). The particles that emerge from the foil will have a residual range equal to \( R_0 - t \). We can then use the range table or graph to determine \( E_1 \) that corresponds to the range \( R_1 = R_0 - t \). The slowing down and averaging of the energy loss rate will be contained in the range function and do not have to be explicitly evaluated.

\textit{Example of range-energy relationship}

Imagine that a beam of \(^{40}\text{Ar}\) ions at 400 MeV (10 MeV/A) is incident on a 18.5 mg/cm\(^2\) beryllium foil (0.1 mm thick). Do the ions pass through the foil and if they do, what is their residual kinetic energy?

Using a standard reference for stopping power, the tables of Northcliffe and Schilling, (At. Data and Nucl. Data Tables A7(1970)233) we find for these ions that dE/dx = 9.597 MeV-cm\(^2\)/mg in beryllium. Thus, for our first estimate of the energy lost:

\[ \Delta E \sim (dE/dx)_{\text{initial}} \Delta x = 9.567 \times 18.5 = 177.5 \text{ MeV} \]

giving a residual energy of \( 400 - 177.5 = 222 \text{ MeV} \) that is almost half the initial kinetic energy. The ions will pass through the foil but this estimate of energy loss is probably too low. Recall that the ions lose more energy per distance traveled as they slow down. Checking we see that the energy loss rate for these ions at 178 MeV is substantially larger, \( i.e., \) dE/dx = 15.3 MeV cm\(^2\)/mg. Thus, this is not a “thin” foil for these ions.
We can use the range technique with information in the same table, for $^{40}\text{Ar}$ ions with $E_0=400$ MeV:

$$R(400 \text{ MeV}) = 28.278 \text{ mg/cm}^2$$

$$R(E_1) = 28.278 - 18.5 = 9.8 \text{ mg/cm}^2$$

This range lies between the tabulated values and by linear interpolation between the range values for $E=160$ and $200$ MeV one finds that $E_1 \sim 185$ MeV. Thus, just using the initial energy loss rate gives a substantial error.

For the practicing nuclear chemist, range-energy tables or relationships are among the most commonly used tools. The largest collection of data on stopping powers and ranges of ions in matter is that of Ziegler and Biersack in the form of the computer programs SRIM/TRIM. Subsets of these tables exist for low energy heavy ions interacting with matter (Northcliffe and Schilling), alpha particles interacting with matter (Williamson, Bijoud and Picard) and for energetic heavy ions (Hubert, et al.).

For the most commonly encountered heavy charged particles, the alpha particles from radioactive decay, some semi-empirical range-energy rules are used. For the range of $\alpha$-particles in air, $R_{\text{air}}$, we have

$$R_{\text{air}} (cm) = (0.005E_{\alpha}(MeV) + 0.285)E_{\alpha}^{3/2}(MeV)$$

or

$$R_{\text{air}} (mg/cm^2) = 0.40E_{\alpha}^{3/2}(MeV)$$
so that the range of a 7 MeV alpha particle in air is about 5.9 cm. For a pure element with $10 < Z < 15$, we have

\[
\frac{R_Z}{R_{\text{air}}} = 0.90 + 0.0275Z + (0.06 - 0.0086Z) \log_{10} \left( \frac{E_\alpha}{4} \right)
\]

where $R_z$ is the range in a pure element of atomic number $Z$ expressed in mg/cm$^2$, $R_{\text{air}}$ is the range in air in mg/cm$^2$, and $E_\alpha$ is the alpha particle energy in MeV. (For $Z < 10$, substitute 1.00 for the term $(0.09 + 0.0275Z)$. For $Z > 15$, replace the term $R_z$ by $(R_{\text{air}} 0.005Z)$). For compounds or mixtures, the range in the compound or mixture, $R_C$ in mg/cm$^2$, is given as

\[
\frac{1}{R_C} = \sum_i \frac{p_i}{R_i}
\]

where $p_i$ is the weight fraction of the $i^{th}$ element in the mixture or compound and $R_i$ is the range of an alpha particle of this energy in the $i^{th}$ element.

**Example**

What is the range of an 8 MeV alpha-particle in air?

\[
R_{\text{air}} (\text{cm}) = (0.005 E_\alpha (\text{MeV}) + 0.285) E_\alpha^{3/2} (\text{MeV})
\]

\[
R_{\text{air}} (\text{cm}) = (0.005 \cdot 8 + 0.285) 8^{3/2} = 7.4 \text{cm}
\]

What is the range of this same alpha-particle in Al?
Application of these formulas show the ranges of decay alpha particles in solids are very short. A sheet of paper will stop the α-particles from most radioactive sources. Alpha emitting nuclei are not external radiation hazards, but because of their high LET they do represent significant inhalation or ingestion hazards.

17.3 Electrons

The passage of energetic electrons through matter is similar to that of heavy charged particles in that the Coulomb interaction plays a dominant role. However, three clear differences can be easily seen: the incident electrons are generally relativistic particles (notice that 1 MeV of kinetic energy corresponds to nearly twice the rest mass of an electron, 0.511 MeV); the scattering is predominantly between identical particles and repulsive; and the interactions with nuclei are attractive and the direction of the electron can be dramatically changed, even reversed, in a collision with a heavy nucleus. A fourth difference that is not so obvious is that a fraction of the kinetic energy is lost through the radiative process of bremsstrahlung. Bremsstrahlung (the German word can be literally translated as "braking radiation") is a general process in which electromagnetic radiation is
emitted whenever a charged particle undergoes a substantial acceleration. The scattering of electrons, particularly to large angles, corresponds to a classical acceleration which creates/requires the emission of bremsstrahlung. By comparison, very few heavy charged particles undergo large accelerations as they slow down in material.

Summarizing this overview of the possible interaction mechanisms for fast electrons in material we find that the rate of energy loss in a material is:

\[
-S\left(\frac{dE}{dx}\right)_{\text{electron}} = S_{\text{electronic}} + S_{\text{radiative}}
\]

where the electronic stopping power is similar to the electronic interaction between charged particles and electrons discussed above whereas the radiative stopping power is specific to electrons. The electronic stopping power for electrons is written as:

\[
S_{\text{electronic}} = -\left(\frac{dE}{dx}\right)_{\text{electronic}} = \frac{2\pi Z e^4 \rho_n}{m_e v^2} \left[ \ln \left( \frac{m_e v^2 E}{2I(1 - \beta^2)} \right) - \ln 2 \left( \frac{\sqrt{1 - \beta^2} - 1 + \beta^2}{(1 - \beta^2) + \frac{1}{8} \left( -\sqrt{1 - \beta^2} \right)} \right) \right]
\]

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using the same definitions as for the stopping power of heavy charged particles. On the other hand, the expression for the radiative stopping power is:

\[
S_{\text{radiative}} = -\left(\frac{dE}{dx}\right)_{\text{radiative}} = \frac{(Z + 1)Ze^4 \rho_n E}{137m_e c^4} \left[ 4 \ln \left( \frac{2E}{m_e c^2} \right) - \frac{4}{3} \right]
\]

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and given that $\beta=1$ contains an extra factor of $ZE$ in the term in front of the parenthesis. With some algebra and for a typical electron energy, one can show that the ratio of the two contributions to the stopping power depends on the atomic number of the material, $Z$, and the electron kinetic energy, $E$:

$$\frac{S_{\text{radiative}}}{S_{\text{electronic}}} \approx \frac{ZE}{800\,MeV}$$

which indicates that the radiative contribution is only significant for large atomic numbers ($Z\sim80-90$) and high electron energies ($E\sim10-100\,MeV$). Typical beta particles from radioactive sources are emitted with only $1-10\,MeV$ of kinetic energy, often much less, and the radiative contribution to the stopping power is very small. The bremsstrahlung spectrum is smooth and continuous ranging from zero energy up to the electron energy due to the random distribution of electron scattering angles.

The important feature that the electrons are occasionally scattered to large angles during the penetration of material causes each electron to follow a tortuous path. A beam of electrons will not have a fixed range in the sense of that a beam of heavy charged particles. In fact, given the identical nature of the particles involved in the scattering process, obtaining the range distribution is problematic. Moreover, the primary or a secondary electron can be scattered backwards and emitted from the material. A schematic plot of the range distribution is shown in figure 17-8 for a monoenergetic source of electrons. The falloff of the intensity with penetration
depth starts immediately as the electron enters the material and then gradually approaches zero. This figure emphasizes the fact that the concept of range cannot be applied in a simple way to energetic electrons. As an approximation, the electron range is taken as the extrapolation of the linear portion of the attenuation curve to zero. As shown in figure 17-9 the product of this definition of the range times the density of a material is a smooth function of incident electron energy for a wide range of materials. Such behavior comes from the fact that the range has a strong dependence on the electron density through the electronic stopping and a weak dependence on the atomic number of the material through the ionization potential and radiative stopping.

Many related measurements have been made of the range distribution of electrons emitted in β⁻ decay. These studies were particularly important before solid-state detectors were available. The measurements have shown that the combination of the Fermi energy distribution of electrons from the decay with the sloping range distribution leads to an approximately exponential attenuation of the beta decay electrons. This can be expressed as

\[ N_t = N_0 e^{-\mu t} \]

where \( N_t \) is the number of beta particles transmitted through a thickness \( t \). The absorption coefficient \( \mu \) can be related to the endpoint energy \( E_{\text{max}} \) of the β spectrum as

\[ \mu \left( \frac{m^2}{kg} \right) = 1.7 \cdot E_{\text{max}}^{-1.14} \]
where $E_{\text{max}}$ is given in MeV.

**Example**

What fraction of the beta particles emitted by $^{32}\text{P}$ ($E_{\text{max}} = 1.71$ MeV) will be stopped by a sheet of Al of thickness 1 mm?

The mass absorption coefficient is $1.7(1.71)^{-1.14} = 0.922$ m$^2$/kg. The fraction of these β particles that are transmitted is $N_t/N_0 = e^{-\mu t} = \exp[-0.922 \text{ m}^2/\text{kg} \times (1 \times 10^{-3} \text{m}) \times (2.7 \times 10^3 \text{kg/m}^3)] = \exp[-2.489] = 0.083$. The fraction absorbed $= 1.0 - 0.083 = 0.917$.

In the distant past, measurements of the attenuation of the beta spectrum from a newly discovered isotope were used to identify the energy of the beta decay. Recently, the attenuation of strong sources has been used to monitor the thickness of materials during manufacturing processes. Notice that the monitoring can be continuous, non-destructive, and a physical probe does not need to "touch" the material being measured.

The backscattering of energetic electrons from materials is a feature not seen with heavy charged particles. Backscattering is primarily due to multiple interactions with (heavy) nuclei that significantly alter the direction of the incident electron that is enhanced by the fact that two energetic electrons can be created when the incident electron scatters from an atomic electron. The coefficient of backscattering is used to quantify the probability that an electron will emerge "backwards" from a surface that is irradiated with electrons. The coefficient is a function of the energy of the incident electron and the atomic number of the absorber. Formally, the coefficient is the fraction of time that an electron is emitted from the surface of a material following the entry of an energetic electron. The coefficient is approximately 0.5 for $E<1$ MeV electrons in gold, approximately 0.3,
0.04 and 0.1 for copper, aluminum, and carbon, respectively. It falls below 0.1 for E=10 MeV electrons in gold, below 0.05 for copper, and to near zero in aluminum and carbon.

The bremsstrahlung radiation from electron beams has important practical applications even though it is a small contribution to the stopping power. Bremsstrahlung forms the basis for operation of x-ray tubes and other "controllable" high fluence sources of radiation. Such devices collide an electron beam with an energy of the order of 10-50 keV with a large electrode, usually made out of a heavy element like tungsten or tantalum. The electrons penetrate the electrode and the bulk of their kinetic energy is lost through electron-scattering and eventually creates heat. However, a small fraction of the incident energy is converted into electromagnetic energy in the x-ray region. This is called "thick target" bremsstrahlung because the incident electrons are completely stopped inside the material. If we assume that the bremsstrahlung is independent of electron energy then the fraction of the electron energy would be $f_{\text{rad}} = 0.0014 Z E$ from the expression above. The observed fraction is about a factor of two lower, which would be consistent with simply taking the average energy of the electron in the material to be 1/2 the initial energy. We should note that some of the energetic electrons can create inner shell atomic vacancies in the atoms that make up the lattice. These vacancies will be filled by discrete K and L x-ray transitions. These sharp lines will add to the continuous bremsstrahlung spectrum and will depend, of course, on the atomic number of the material.
Another mechanism for electron energy loss in matter is the emission of Cerenkov radiation. When a beam of fast moving charged particles with a velocity \( v \) near the speed of light \( c \) enters another medium with index of refraction \( n \), the particle velocity will exceed the speed of light in the new medium (which is \( c/n \)). The electron radiates the "excess" energy in the form of a blue-white light called Cerenkov radiation. This light is localized in a cone of half-angle \( \theta \) around the direction of motion of the electron such that \( \cos \theta = c/nv \).

17.4 Electromagnetic Radiation

As photons move through material they only interact or "scatter" in localized or discrete interactions and they do not interact at long distances, *i.e.*, they are not subject to the Coulomb or nuclear forces. This behavior is in clear contrast to the long-range interactions felt by charged particles. Thus, as a beam of photons propagates through any material, the *intensity* of the beam will decrease as the photons that interact are removed but the *energy* of all the non-interacting photons will remain constant. The photons will interact in ways that predominantly release fast moving electrons, low energy photons will interact only once and give rise to a single primary electron, energetic photons can interact several times and give rise to a few primary electrons. The most energetic photons can create a matter-antimatter pair of electrons that induce a cascade of secondary electrons.

The energy of the non-interacting photons remains constant so that the probability that a photon will interact in a fixed thickness of material will also
remain constant regardless of the photon energy. This leads immediately to an exponential attenuation of electromagnetic radiation that is called the Beer-Lambert Law. The law was applied to the absorption of visible light but applies to all electromagnetic radiation. The derivation of the exponential attenuation law is similar to the derivation of the exponential decay law of radioactive nuclei and will not be repeated here. (The analogy is that the probability of radioactive decay is constant in a given time interval.)

The general expression for the attenuation of photons is:

$$I = I_0 e^{-\mu x}$$

where $I$ and $I_0$ are transmitted and incident intensities, respectively, $x$ is the thickness, and $\mu$ is an energy dependent total linear absorption coefficient that depends on the nature of the material. The mean free path of the photons in the material is simply $\lambda=1/\mu$ and we can define a half-thickness as $x_{1/2} = \ln(2)/\mu$ in analogy to the radioactive half-life. The exponential nature of the attenuation means that the intensity of the transmitted radiation does not go to zero, although it can be made arbitrarily small. The mass attenuation coefficient is obtained by dividing the linear attenuation coefficient by the density of the material, $\mu/\rho$. The mass attenuation coefficient is independent of the physical state of the absorber and represents the fact that the fundamental interactions can be expressed in terms of cross sections per atom. Extensive tabulations and figures, such as figure 17-10, are available for the mass attenuation coefficients of photons with energies in the range of 0.01 to 10 MeV.
Example of Photon Attenuation

Estimate the fraction of 1.0 MeV photons that will be transmitted through a lead absorber that is 5 cm thick (the thickness of "lead bricks" commonly used in radiation shields).

The transmitted fraction is simply:

\[ f = \frac{I}{I_0} = e^{-\mu x} \]

where \( x \) is 5 cm and \( \mu \) can be obtained from the mass attenuation coefficient in figure 17-10. Reading the value from figure 17-10, \( \mu_0/\rho = 0.07 \text{ cm}^2/\text{g} \) for the total attenuation. \( \mu_0 \) is equal to \( \mu_0/\rho \ast (\rho) \), of course. Thus:

\[ f = \frac{I}{I_0} = e^{-(\mu_0/\rho)x} \]

\[ f = e^{-(0.07 \text{ cm}^2/\text{g}) \ast 11.35 \text{ g/cm}^3 \ast 5 \text{ cm}} = e^{0.795 \ast 5} = 0.019 \]

Approximately 2% will be transmitted. Notice that the half-thickness for these photons in lead, \( x_{1/2} = \ln 2/\mu \) is 0.87 cm.

Concentrating on photon energies that are associated with nuclear energy levels, those in the region from 10 keV to 10 MeV, we find that only three types of interactions play a role in attenuating a photon beam. These mechanisms are shown schematically in Figure 17-11. Each photon that interacts with an atom via any one of the mechanisms will be lost from beam. The type of interaction is random but their relative probabilities depend on the photon energy. Photoelectric absorption dominates at low energies, \( E < 0.1 \text{ MeV} \) in the heaviest elements, while Compton scattering is most important at intermediate energies, whereas pair-production has an absolute threshold at 1.022 MeV and only is important for the highest energy photons. We will consider each of these processes in turn.
4.1 Photoelectric Effect

The photoelectric effect was originally described by Einstein and helped to establish the quantized nature of light. The photoelectric effect has many extremely important applications for example the detection of visible light by photocells and the photovoltaic conversion of sunlight. The photoelectric effect converts a single photon into a single free electron. When the photon interacts with a bound electron, the photon can be completely absorbed and the electron emerges with a kinetic energy that corresponds to the photon energy, $h\nu$, minus the electron binding energy, $KE_e = h\nu - BE$ (see Figure 17-11). Photocells use a semiconductor like silicon for the absorbing material and the electrons released by visible light have relatively small kinetic energies and are collected as a photocurrent. In the present application of the absorption of a nuclear photon, a fast electron is usually created in the bulk of a solid medium because the binding energy is often small compared to the photon energy. The fast electron goes on to lose its kinetic energy by scattering through the material (as discussed above). Conservation of momentum requires that the electron be bound in an atom (that could be in a lattice) that recoils.

The cross section or probability of the photoelectric effect is on the order of the square of the atomic size for photons in the keV region and decreases rapidly with increasing photon energy. The cross section also has a strong dependence on the atomic number of the absorbing material, as there is a sharp increase in the cross section at each threshold for the emission of bound electrons. As an example, the heavy element lead ($Z=82$) has K, L and M (principal quantum numbers, $n=1,2,$ and
3) binding energies of approximately 88, 15, and 3 keV, respectively, which provide strong photoelectric absorption for photons in this energy region. The sharp increase in the photoelectric cross section can be seen as the sharp peaks on the left in figure 17-10. The overall probability for photoelectric absorption follows the very rough expression:

$$\sigma_{\text{photoelectric}} \propto Z^n / E^7 \gamma$$

where the exponent, n, is between 4 and 5. This expression only includes the dramatic effects of the electron binding energies in an overall way and is not meant to replace the measured values.

We should note that the photoelectric effect often leaves an inner shell vacancy in the atom that previously contained the "ejected" electron. This vacancy will be filled by an atomic transition, called fluorescence, and generally produce an x-ray photon. In an interesting twist of fate, the x-ray photon will have an energy that is just below the sharp rise in the attenuation coefficient due to conservation of momentum and can often escape from the absorber. Recall that the direction of the fluorescence photon will be uncorrelated with the direction of the incident photon and a fraction will be emitted "backwards" from the absorber. The absorber will thus emit its' own characteristic x-rays when it is irradiated with high energy photons.

In $\gamma$-ray spectroscopy lead shields are commonly used. This can result in the production of Pb x-rays that can interfere with the measurement of low energy
photons. Lining the Pb shields with layers of Al and Cu that absorb the Pb x-rays and other subsequent radiation ameliorates these problems.

4.1 Compton Scattering

If the energy of the incident photon exceeds the typical binding energies of the innermost atomic electrons, the probability of photoelectric absorption drops below the probability that the photon will scatter from an electron leading to a scattered electron and a photon. This process is called Compton scattering. A schematic diagram of this process is shown in Figure 17-12.

From the conservation of momentum in the x direction we have

\[ p = p' \cos \theta + p_e \cos \phi \]

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From conservation of momentum in the y direction, we have

\[ 0 = -p' \sin \theta + p_e \sin \phi \]

17-34

Assuming the collision is elastic, conservation of energy gives us

\[ E = E' + T_e \]

17-35

Combining these equations (see problems) and utilizing the relativistically correct expressions for the energy and momentum of the electron

\[ p_e^2 = \frac{1}{c^2} \left[ T_e \left( T_e + 2mc^2 \right) \right] \]

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we get
\[ \lambda' - \lambda = \frac{\hbar}{m_e c} \left(1 - \cos \theta\right) \]

where \( \lambda' \) and \( \lambda \) are the wave lengths of the scattered and incident \( \gamma \)-rays. The quantity \( \frac{\hbar}{m_e c} \) is called the Compton wave length of the electron and is equal to 2426 fm. Note that the shift in \( \gamma \)-ray energy is independent of the incident energy. The expressions for the energies of the scattered photon and electron are given in Table 17-3.

It is clear that the minimum energy of the scattered \( \gamma \)-ray occurs when \( \theta = 180^\circ \) \((\cos \theta = -1)\). In this case, we have

\[
E_{\gamma'}^{\text{min}} = \frac{m_e c^2}{2} \left( \frac{1}{1 + \frac{m_e c^2}{2E_{\gamma}}} \right) \approx 255\text{keV}
\]

In this case, the electron energy, \( T_e \) will be maximum and \( T_e = E_{\gamma} - 0.255\text{MeV} \). If we consider all scattering angles \( \theta \), then the distribution of scattered electron kinetic energies is as shown in Figure 17-13. The sharp peak at \( E_{\gamma} - 0.255\text{MeV} \) is called the Compton edge. The minimum energy photon, \( E \approx 255\text{keV} \), will be a noticeable component in \( \gamma \)-ray spectra resulting from the interaction of a photon from a radioactive source with the lead shield surrounding the detector, resulting in a
backscattered photon ($E \sim 255\text{keV}$) which strikes the radiation detector (the “backscatter peak”).

The Compton scattering cross section per electron of the stopping material is independent of $Z$, and thus the cross section per atom goes as $Z$. For energies about 0.5MeV, it varies roughly as $1/E$.

### 4.2 Pair Production

Whenever the energy of the initial photon exceeds the rest mass of two electrons, 1.022 MeV, the process of pair production is possible. During the process of pair production, the initial photon interacts with the Coulomb field of a nucleus and is converted into an electron and a positron, a matter-antimatter pair, that shares the initial energy of the photon (see Figure 17-11). Conservation of energy and momentum in the Coulomb field cause the pair of electrons to move forward along the initial direction of the photon with a small opening angle. The pair of particles will interact with the electrons and nuclei in the remaining material as described above.

The process of *bremsstrahlung* observed in electron stopping is closely related to the process of pair production. From a schematic standpoint, in the first case, a moving electron interacts with the Coulomb field of an atom, making a transition between two energy states and a (bremsstrahlung) photon is emitted. In the second case, a photon is destroyed by interaction with an atomic Coulomb field and a pair of electrons is created. The probability of pair production has an absolute threshold of 1.022 MeV, that is, this process cannot take place if the
photon has a lower energy. The cross section increases relatively rapidly and saturates above ~10 MeV, as indicated in figure 17. The variation of the pair-production cross section with photon energy is complicated but the cross section depends on the square of the atomic number of the absorber. For large photon energies, \( \sigma_{\text{pair}} \propto Z^2 \ln (E\gamma/mc^2) \). Pair production is the predominant attenuation process for high-energy photons.

Pair production has a threshold energy of 1.022 MeV because two particles are created, one electron and one positron. Thus some energy is ‘stored in’ or ‘used to create’ the mass of the pair. Notice the total electric charge is conserved because the electron charge is -1 e and the positron charge is +1 e. One of the unique features of this process is that the energy that went into the creation of the two particles will be ‘released’ when the positron comes to rest and annihilates with an electron. The annihilation process is:

\[ e^+ + e^- \rightarrow \gamma + \gamma \]

in which the two gamma rays have exactly the same energy, \( mc^2=0.511 \text{ MeV} \), and are emitted at 180°, or back-to-back. The axis along which the two gamma rays are emitted will be random with respect to the initial direction of the incident photon because the positron will undergo a slowing down process involving multiple scattering with atoms and atomic electrons. Therefore, the characteristic annihilation radiation (photons with an energy of 0.511 MeV) can escape from the absorber whenever pair production can occur.

In summary, photons pass through material until they interact individually with the atoms or nuclei in the material. Depending on the energy of the photons the
interaction will be predominately pair-production (high energy), Compton scattering, or photoelectric absorption (low energy). The relative importance of these processes is summarized in figure 17-14 as a function of atomic number and photon energy. In each interaction, the photon ionizes the material creating one or two fast moving electrons and leaving a positive ion. Pair production gives two fast moving electrons - one positive, one negative, Compton scattering gives one fast moving electron and a lower energy photon, and the photoelectric effect gives one fast moving electron. The fast electrons have a much higher rate of ionization than the photons and the general features of the interactions of these electrons with the material have been described above. The ‘path’ of the scattered photon will be erratic when Compton scattering is the predominant process. Gamma rays from nuclear decay processes tend to have energies on the order of 1 MeV. From figure 17-14 we expect these photons to interact via Compton scattering in all materials. The first interaction will give an electron and a lower energy photon. The interaction probability of the secondary photon will usually be higher that that of the primary photon and will often result in a photoelectric absorption. Recall also that, depending on the energy of the primary photon, the absorber will weakly emit lower energy photons such as annihilation radiation, backscatter radiation or fluorescence x-rays.

5. Neutrons

We will now briefly consider the propagation of neutrons through material. Neutrons are the most penetrating radiation for the simple reason that their only
significant interaction is with nuclei via the strong force. (Neutrons only have a very small interaction with electrons through their magnetic dipole moment that can be ignored.) As we discussed at the beginning of this chapter, nuclei are very much smaller than atoms and so the probability that a fast neutron will interact with (strike) a nucleus is very, very small. On the other hand, neutrons cause significant radiation damage because all of their interactions cause nuclear recoil and many lead to nuclear transmutations.

A neutron will move through material along a straight line with a constant energy until it encounters a nucleus and induces a nuclear reaction. Thus, neutron attenuation follows an exponential law similar to that for photons. Written in terms of an energy dependent attenuation length, \( \mu_E \) we have:

\[
I = I_0 e^{-\mu_E x}
\]

17-39

where \( x \) is a linear dimension and \( I_0 \) is the incident intensity. The attenuation length is the inverse of the mean free path, \( \lambda \), and is proportional to the total nuclear reaction cross section:

\[
\mu_E = \frac{1}{\lambda_E} = N_0 \sigma_{Total}(E)
\]

17-40

\( N_0 \) is a constant that gives the total number of nuclei per unit volume in the material. The total nuclear reaction cross section is a characteristic of each isotope in the absorbing material and has the dimensions of an area. If we have a monoisotopic element such as gold or bismuth, then we will only have to account
for the energy dependence of the neutron. If the material contains several isotopes such as silver ($^{107}\text{Ag}$ and $^{109}\text{Ag}$), nickel (five isotopes), or is a compound NaF (one isotope of each element), etc., then the effective cross section will be the *number-weighted* cross section:

\[
\sigma_{\text{average}} = (f_1 \sigma_{\text{Total}(E)_1} + f_2 \sigma_{\text{Total}(E)_2} + f_3 \sigma_{\text{Total}(E)_3} + \ldots )
\]

where the constants, $f_i$, are the fraction by number of each isotope in the sample.

*Example of an average nuclear cross section for a compound*

Calculate the average thermal neutron capture cross section and the mean free path for LiF, a solid crystalline material at room temperature with a density of 2.635 g/cm$^3$ and a molar mass of 25.94 g/mole. Lithium has two stable isotopes $^6\text{Li}$ (7.5%) and $^7\text{Li}$ (92.5%) with thermal neutron capture cross sections of $\sigma_{\text{thermal}}=39$ mb and 45 mb, respectively. Fluorine is monoisotopic, $^{19}\text{F}$, with $\sigma_{\text{thermal}}=9.6$ mb.

\[
\sigma_{\text{average}} = (0.075 \times 0.5 \times 39 \text{ mb} + 0.925 \times 0.5 \times 45 \text{ mb} + 1.0 \times 0.5 \times 9.6 \text{ mb})
\]

\[
\sigma_{\text{average}} = 27.1 \text{ mb}
\]

Rearranging the equation above relating the mean free path and the total reaction cross one has:

\[
\lambda_{\text{thermal}} = \frac{1}{N_0 \sigma_{\text{thermal}}}
\]

\[
N_0 = N_A \rho / \text{Molar Mass} = \frac{6.022 \times 10^{23} \text{ mol}^{-1} \times 2.634 \text{ g/cm}^3}{25.94 \text{ g/mol}}
\]

\[
N_0 = 6.11 \times 10^{22} \text{ cm}^{-3}
\]

And finally for the mean free path:
\[
\lambda_{\text{thermal}} = \frac{1}{6.11 \times 10^{22} \text{ cm}^{-3} \times 27.1 \text{ mb} \times 1 \times 10^{-27} \text{ cm}^2 / \text{ mb}}
\]

\[
\Delta_{\text{thermal}} = 604 \text{ cm}
\]

Thus, the average thermal neutron travels more than six meters in solid LiF before undergoing a nuclear capture reaction! Note the total reaction cross section will be larger and the neutrons will most likely scatter before being captured.

Neutrons can interact with matter via a number of different reactions, depending on their energy. Among the most important of these reactions are:

(a) elastic scattering, \( A(n,n)A \), which is the principal interaction mechanism for neutrons.

(b) inelastic scattering, \( A(n,n')A^* \), where the product nucleus \( A^* \) is left in an excited state. To undergo inelastic scattering, the incident neutron must have sufficient energy to excite the product nucleus, generally about 1 MeV or more.

(c) radiative capture, \( A(n,\gamma)A+1 \). As discussed earlier, this cross section shows a \( 1/\nu \) energy dependence and this process is important for low energy neutrons.

(d) fission, \( A(n,f) \), which is most likely at thermal energies but occurs at all energies where the neutron binding energy exceeds the fission barrier height for fissile nuclei.

(e) knockout reactions, such as \( (n,p) \), \( (n,\alpha) \), \( (n,t) \), etc. which are maximum for neutrons of eV-keV energy but occur at higher energies.

The total neutron interaction cross section, \( \sigma_{\text{total}}(E) \), is the sum of the various reaction cross sections.
One of the technologically most important interactions of neutrons with matter is their loss of energy ("slowing down") by a series of elastic collisions. Let’s consider the case where particle (1) of mass $m_1$, speed $v_1$, collides with particle (2), mass $m_2$, at rest. After the collision, the particles will have speeds $v_1'$ and $v_2'$ in the lab system.

In the center of mass (cm) system, we have

After the collision, the relationship between the cm and the lab systems is

$$\sigma_{\text{total}} = \sigma_{\text{elastic}} + \sigma_{\text{inelastic}} + \sigma_{\text{capture}} + \cdots$$
If we designate the kinetic energy of particle (1) after the collision at $T_{1}^\prime$, we have

$$T_{1}^\prime = \frac{1}{2} m_{1} \left( v_{1}^\prime \right)^{2}$$

$$T_{1}^\prime = \frac{1}{2} m_{1} \left( v_{1} + v_{0} \right)^{2} = \frac{1}{2} m_{1} \left( v_{1}^{2} + v_{0}^{2} + 2 v_{1} v_{0} \cos \theta \right)$$

Thus $T_{1}^\prime$ will have a maximum value for $\theta = 0^\circ$ and a minimum value for $\theta = 180^\circ$.

We have

$$T_{1}^\prime (\text{max}) = \frac{1}{2} m_{1} \left( v_{1}^{2} + v_{0}^{2} + 2 v_{1} v_{0} \right) = \frac{1}{2} m_{1} v_{1}^{2} = T_{1}$$

$$T_{1}^\prime (\text{min}) = \frac{1}{2} m_{1} \left( v_{1}^{2} + v_{0}^{2} - 2 v_{1} v_{0} \right) = \frac{1}{2} m_{1} \left( v_{1} - v_{0} \right)^{2}$$

$$= \frac{1}{2} m_{1} \left( v_{1} - 2 v_{0} \right)^{2}$$

$$= T_{1} \left( \frac{m_{1} - m_{2}}{m_{1} + m_{2}} \right)^{2}$$
For the special case where particle (1) is a neutron and particle (2) is a proton, $m_1 = m_2$

$$T'_1(\text{min}) = 0$$

$$T'_1(\text{max}) = T_1$$

It follows that

$$T'_1 \frac{T_1 (1 + \cos \theta)}{2}$$

$$\theta_1 + \theta_2 = 90^\circ$$

$$v_1 = v_2 = v_0$$

If we assume the angular distribution of the scattered neutrons is isotropic in the cm system, then the probability of a neutron scattering into and solid angle $d\Omega$, $P(d\Omega)$, is a constant given by

$$P(d\Omega) = \frac{d\Omega}{4\pi}$$

where the solid angle $d\Omega$ is given in steradians. Substituting in for $d\Omega$, assuming spherical symmetry we have

$$P(d\Omega) = \frac{2\pi \sin \theta d\theta}{4\pi}$$

$$= \frac{1}{2} \sin \theta d\theta$$
When a neutron is scattered into an angular interval \((\theta to \theta + d\theta)\), its energy is changed from \(T_1\) to the interval \((T_1' to T_1' + dT_1')\). Here

\[
dT_1' = -m_i V_i v_0 \sin \theta d\theta
\]

Thus we have for the probability of scattering into an energy interval \(dT_1'\)

\[
P(dT_1') = P(d\Omega) = \frac{1}{2} \sin \theta d\theta = \frac{dT_1'}{2m_i V_i v_0}
\]

We have an equal probability of scattering into each energy interval. For neutrons scattering off hydrogen

\[
T_1'(\text{min}) = 0
\]

\[
T_1'(\text{average}) = \frac{T_1}{2}
\]

After \(n\) collisions

\[
T_1'(\text{average}) = \left(\frac{1}{2}\right)^n T_1
\]

Thus, to reduce a 1 MeV neutron to thermal energies (~1/40 eV) would require about 25 collisions.
6. Radiation Exposure and Dosimetry

Up to this point we have taken a very microscopic view of the propagation of beams of particles through material. We have described the degradation of the intensity and the energies of the beams in terms of individual interactions. Now we will take a more macroscopic view from the standpoint of the absorber.

As we have seen the passage of radiation through material causes ionization of the atoms and molecules. The creation of free electrons, recoiling positive ions and in some cases transmuted nuclei can disrupt the chemical structure of the material. It is important to note that the effect of the passage of a single particle through a macroscopic object will usually cause a minimal overall effect due to the enormous number of atoms present in the object. Special devices are necessary to observe the ionization from individual particles as described in the next chapter on radiation detectors. Physical changes in an everyday object will be observed when it has been exposed to large amounts of radiation. At the same time we should realize that certain materials will be relatively immune to the ionization caused by photons and fast electrons, whereas neutron irradiation of the same material generally will have a substantial effect. For example, a metal lattice is characterized by delocalized electrons and the local ionization caused by a Compton scattering or photoelectric absorption will be quickly neutralized (or repaired). Neutron absorption by a metal will generally lead to beta-decay and the transmutation of one atom into the neighboring element. On the other hand, local ionization created by photons and electrons in insulating materials like glasses will persist for a long
time. Similarly, lattice defects caused by atomic recoil have to be removed by annealing but atomic recoil has little effect in liquids (and none in gases). Thus we can see that the effects of radiation on a material will depend in great detail on the type and amount of radiation, and on the physical and chemical nature of the material being irradiated.

We have seen that the neutral forms of radiation, photons and neutrons, are very penetrating and can pass through layers of material without interacting. In these cases, we need to distinguish between the amount of radiation exposure and the amount of energy absorbed by the material. Photons such as x-rays, bremsstrahlung, and gamma rays play an important role in nuclear medicine but they are not strongly absorbed by tissue. The exposure to these photons is not equal to the dose. In the case of highly ionizing radiation such as charged particles, the exposure will correspond to the absorbed energy except for very thin materials that allow the particle to escape. In order to characterize radiation effects, we need to know the amount of energy absorbed by the material, which is called the absorbed dose.

The unit of radiation exposure is the roentgen (R). It is a historical unit of the exposure and characterizes the radiation incident on an absorbing material without regard to the character of the absorber. The unit was formalized in 1928 as "The amount of radiation which produces one electrostatic unit of ions (esu), either positive or negative, per cubic centimeter of air at standard temperature and pressure." Translated in modern units:

\[ 1 \text{ Roentgen} = 2.58 \times 10^{-4} \text{ Coulomb/kg-air} = 0.3336 \text{ nC/cm}^3 \text{ at STP} \]
This value corresponds to an absorbed energy of approximately 8.8 ml/kg using the effective ionization energy of 34 eV per ion pair in air. The roentgen is most often used to describe the intensity of a photon source such as a medical x-ray machine or other irradiator. The exposure should be measured at some distance from the source so that the radiation field is uniform compared to the dimensions of the detector. The detector is usually an ion chamber filled with dry air that is sensitive to pico-Coulombs of charge.

As studies of the effects of exposure to all types of radiation went on, it became clear that these effects were correlated with the amount of absorbed energy, which is generally less than the exposure. In 1962 the “rad” was formally defined as a special unit of energy called the "Radiation Absorbed Dose" with a value of 100 ergs per gram of absorbing material. The rad is a convenient physical standard that correlates well with chemical and biological effects of radiation whereas the roentgen defined in terms of an air ionization measurement was left for exposure. More recently, the Gray (Gy) was introduced as the SI unit for the absorbed dose:

\[ 1 \text{ Gy} = 1 \text{ J/kg} = 100 \text{ rad} = 6.24 \times 10^{12} \text{ MeV/kg} \]

so that one centigray is exactly one rad. One Gray corresponds to a relatively large amount of energy to be absorbed from a radiation source per unit mass. For example, 1 Gy in water is 18 ml/mole. Modern dosimeters routinely measure doses at the few millirad (mr) level or few tens of microGy

The acronym “kerma” for "Kinetic Energy Released in absorbing Material" has been used to conceptually connect the energy deposited by ionizing radiation with
the radiation field. It is defined to include the kinetic energy, which is locally absorbed from products of interaction with the particular medium such as Compton electrons, photoelectrons, and pair production while excluding the energy, which is not locally absorbed, from Compton-scattered photons, characteristic fluorescence radiation, and annihilation photons. The kerma is defined as:

\[ K = \phi E \mu_\text{k} / \rho \]

where \( \phi \) is the particle fluence (m\(^{-2}\)), \( E \) is the energy of the radiation, \( \mu_\text{k} \) is a linear energy attenuation coefficient (energy/m), and \( \rho \) is the density (kg/m\(^3\)). The dimensions of the kerma are thus J/kg or Gray. The concept of kerma in air is very close to the practical definition of the roentgen as a unit of exposure, times a factor for the amount of energy necessary to create an ion pair. However, bremsstrahlung photons are lost in the secondary-particle equilibrium condition assumed in the definition of the roentgen but they represent a small effect under most conditions.

Just as the effect of radiation on a specific material depends on the dose or amount of absorbed amount of energy in contrast to the exposure, the effect of radiation on biological systems depends on the energy density and not just the energy. One can imagine that a biological system could survive the formation of a single ion pair and the following chemical transformations from a photoelectric event. However, if a large number of direct ionization events take place in a small volume due to the passage of a heavy charged particle through some biological material, the resulting chemical changes could be profound. The important parameter is called the "linear energy transfer" or LET which is very close to the
specific energy loss, \(-dE/dx\), discussed earlier in this chapter. The value of LET for a given particle is smaller than the value of \(-dE/dx\) because the LET does not include the radiative energy loss term, as the bremsstrahlung radiation is not absorbed "locally." Recall that the radiative energy-loss term was only significant for high-energy electrons. Typical values of LET for photons and fast electrons are a few MeV/mm but are one or two orders of magnitude larger for heavy charged particles.

The concept of dose-equivalent has developed over time to quantify the more damaging effects of high LET radiation. The original definition of the absorbed dose in rads was multiplied by a quality factor, Q > 1. The quality factor increased with increasing LET. The historical unit for dose-equivalent is called a rem for "roentgen equivalent man" and measurements of dose equivalents to biological systems, especially people, are most commonly reported in millirem (mrem). With the more recent SI dose unit of Gray, a new SI unit of dose-equivalent (or as it is now called, equivalent dose) was introduced called the Sievert (Sv). The Sievert and rem are different by a factor of one hundred in the same way as the rad and Gray:

\[ 1 \text{ Sv} = 100 \text{ rem} \text{ or } 10 \mu \text{Sv} = 1 \text{ mrem} \]

The equivalent dose in Sv = absorbed dose in grays \( \times w_R \) (radiation weighting factor, formerly the quality factor). The absorbed dose for low LET radiation, beta and gamma rays, is taken as having a radiation weighting factor of unity, \( w_R =1 \). The radiation weighting factor has been defined to increase in proportion to the log of the LET. Thus, the radiation weighting factor for alpha particles in tissue is
about 20. The factor for neutrons takes an intermediate value due to the high probability for scattering protons in tissue. A listing of radiation weighting factor values for various types of radiation is shown in Table 17-4.

Notice that the dose has a strict definition of energy per unit mass of the absorber and, in principle, can be measured for a given radiation at a certain energy in a specific material. The equivalent dose is a relative unit in that a radiation weighting factor is applied to a measured quantity. The dose can be measured from ionization in an electronic radiation detector; the equivalent dose must take into account the type of radiation causing the ionization.

References


   An important, easy to read summary of this subject.


4. K. H. Lieser, Nuclear and Radiochemistry: Fundamentals and Applications (VCH, New York, 1997) Covers a number of the practical aspects of the subject that are important to radiochemists.


9. J. Ziegler and J.P. Biersack, [http://www.srim.org](http://www.srim.org) This website and the references cited therein represent the largest and most widely used compilation about the stopping of energetic ions in matter. The computer programs SRIM and TRIM found here are used widely to estimate stopping powers, ranges and straggling.


11. F. Hubert, R. Bimbot, and H. Gauvin, At. Data and Nucl. Data Tables 46, 1 (1990). This reference contains ranges and stopping powers for the interaction of 2.5- 100 MeV/nucleon heavy ions with matter.
Problems.


2. At what kinetic energy does an electron have the same energy loss as a 6 MeV alpha particle interacting with aluminum?

3. Calculate $dE/dx$ for an 8 MeV alpha particle interacting with Mylar.

4. It has been said very approximately alpha particles and protons having the same speed have approximately the same range in matter. Why is this false? Which has the longer range and why?

5. Verify that the minimum ionization for heavy charged particles takes place at $\beta \sim 0.96$.

6. Thin nickel foil is used to slow down monoenergetic 10 MeV protons. What is the maximum thickness that can be used if one wants the straggling to be < 1% of the mean transmitted energy? What is the mean transmitted energy in this case?

7. Calculate the thickness of aluminum foil needed to degrade a beam of 10 MeV/nucleon $^{12}$C ions to 3 MeV/nucleon.

8. Calculate the energy loss of a 6 MeV alpha particle in passing through 50 $\mu$g/cm$^2$ of natural nickel?

9. Assuming no energy losses occur, calculate the heating of a 500 $\mu$g/cm$^2$ foil of $^{208}$Pb when bombarded with 1 particle microampere of $^{86}$Kr$^{19+}$ ions.

10. Devise a way, using measurements of $dE/dx$ and E, to build a particle identification system.
11. Calculate the range in aluminum of: (a) an 80 MeV $^{80}$Br ion (b) a 12 MeV alpha particle and (c) a 1 MeV electron.

12. A GM counter window is made of mica, NaAl$_3$Si$_2$O$_{10}$(OH)$_2$ with a thickness of 2 mg/cm$^2$. (a) What is the minimum energy $\beta$-particle that can penetrate this window? (b) What is the minimum energy alpha particle that can penetrate this window?

13. Repeat the calculation outlined in problem 12 for skin of thickness 1 mm with an average density of 1g/cm$^3$. Assume skin is 65% O, 18%C, 10%H and 7% N.

14. Particles of kinetic energy 400 MeV are incident on a medium of index of refraction of 1.888. One observes Cerenkov radiation with an opening angle of $\theta = \cos^{-1}(0.55)$. What are the particles?

15. How far does a $^{32}$P $\beta$-particle ($E_{\text{max}} = 1.7$ MeV) penetrate in P-10 counter gas?

16. Suppose you have a sample that contains radionuclides that emit 1 MeV $\beta$-particles and 1 MeV $\gamma$-rays. Devise an attenuation technique that would allow you to count the $\gamma$-rays without interference from the $\beta$-particles.

17. What is the fractional attenuation of a beam of 1 MeV photons in 2.5 cm of Pb?

18. Prove that a photon with $E > 1.022$ MeV cannot undergo pair production in free space.

19. Lead is thought to be a “better” absorber of photons than aluminum. At what gamma ray energies is the mass absorption coefficient of lead greater than that of aluminum? Why?
20. A 1 MeV photon undergoes Compton scattering through angles of 0°, 90°, and 180°. What is the energy of the scattered photon in each case?

21. What is the mean free path of a 0.1, a 1.0 and a 3.0 MeV photon in NaI?

22. Calculate the mean free path of a 200 keV photon in water.

23. How much lead shielding will it take to reduce the radiation exposure level to < 10 mrem/h 1 foot from a 5 mCi $^{60}$Co source?

24. Prove the scattering angle is 90° for $A + A$ elastic scattering.

25. Consider a particle with mass $m_1$ scattering elastically from a particle (at rest) with mass $m_2$. If $m_1 > m_2$, show that the scattering angle cannot exceed $\sin^{-1}(m_2/m_1)$.

26. In graphite, how many collisions are necessary to reduce the kinetic energy of a 1 MeV neutron to thermal energies? What is the approximate time scale for this process?

### Tables

Table 1: Particle Classes for Interaction with Matter

<table>
<thead>
<tr>
<th>Charged Particles</th>
<th>Uncharged Particles</th>
</tr>
</thead>
<tbody>
<tr>
<td>protons, heavy ions</td>
<td>neutrons</td>
</tr>
<tr>
<td>electrons</td>
<td>gamma rays</td>
</tr>
</tbody>
</table>
Table 2: Electron Densities

<table>
<thead>
<tr>
<th>Element</th>
<th>Atomic Number</th>
<th>Density (g/cm³)</th>
<th>Atomic Mass (g/mol)</th>
<th>$\rho_N (N_A/cm^3)$</th>
<th>$\rho_e = Z \rho_N (N_A/cm^3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beryllium</td>
<td>4</td>
<td>1.85</td>
<td>9.0122</td>
<td>0.205</td>
<td>0.821</td>
</tr>
<tr>
<td>Aluminum</td>
<td>13</td>
<td>2.70</td>
<td>26.98</td>
<td>0.10</td>
<td>1.3</td>
</tr>
<tr>
<td>Silver</td>
<td>47</td>
<td>10.5</td>
<td>107.88</td>
<td>0.0973</td>
<td>4.6</td>
</tr>
<tr>
<td>Gold</td>
<td>79</td>
<td>19.3</td>
<td>197.0</td>
<td>0.0980</td>
<td>7.7</td>
</tr>
</tbody>
</table>
Table 3: Compton Scattering at fixed angles.

<table>
<thead>
<tr>
<th>$\theta \gamma$ (degrees)</th>
<th>Emitted Photon Energy $(h\nu')$</th>
<th>Electron Kinetic Energy $(\sim h\nu - m_e c^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$h\nu$</td>
<td>$\sim$ zero</td>
</tr>
<tr>
<td>90</td>
<td>$h\nu \left( \frac{m_e c^2}{m_e c^2 + h\nu} \right)$</td>
<td>$\sim h\nu - m_e c^2$</td>
</tr>
<tr>
<td>180</td>
<td>$h\nu h\nu \left( \frac{m_e c^2}{m_e c^2 + 2h\nu} \right)$</td>
<td>$\sim h\nu - m_e c^2/2$</td>
</tr>
</tbody>
</table>

Table 4. Radiation weighting factors for various radiations

<table>
<thead>
<tr>
<th>Radiation</th>
<th>$\gamma$</th>
<th>$\beta$</th>
<th>proton</th>
<th>$\alpha$</th>
<th>fast n</th>
<th>thermal n</th>
</tr>
</thead>
<tbody>
<tr>
<td>weight</td>
<td>(&gt; 2 MeV)</td>
<td>(2-20 MeV)</td>
<td>Radiation</td>
<td></td>
<td>weight factor</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figures

Figure 17-1 The trails of ionization left by heavy charged particles (initial energy of 10 MeV/nucleon) as they penetrate through a photographic plate (nuclear emulsion). The ions interact with the atomic electrons in the emulsion creating ion pairs to "expose" the emulsion and the tracks become visible after the film is developed. Notice the straight-line tracks. [From Knoll]

Figure 17-2 Trajectory of a moving ion past an electron. [From Leo]
Figure 17-3 The energy loss rate as a function of thickness for a $^{40}$Ar projectile in beryllium metal is shown on the scale to the right for an ion that enters the foil at the very high energy of 8 GeV. The remaining kinetic energy of the ion is shown on the left scale. Note that the ion penetrates approximately 21 mm into the metal.
Figure 17-4. The density of ionization along the path of an alpha particle stopping in air is shown. The Bragg peak in the ionization density is evident. M.G. Hollaway and M.S. Livingston, Phys. Rev, 54, 29 (1938).

Figure 17-5. Energy spectrum of 3 MeV protons (a) before and (b) after passing through a 3.3 mg/cm² gold foil. From L.P. Nielson, Dan. Mat. Fys. Medd. 33, No. 6 (1961).
Figure 17-6 The intensity distribution or attenuation curve is shown as a function of absorber thickness for a typical energetic heavy-ion penetrating into a metal. The effect of range straggling is indicated by the Gaussian distribution of ranges. [From Leo]
Figure 17-7 The range-energy curves for some charged particles in silicon. Note the data has the form $R = aE^b$ with a similar exponent for all ions. [From Knoll]

Figure 17-8 A schematic attenuation curve for an energetic electron in a solid material. [From Knoll]
Figure 17-9. The product of the range of an energetic electron in a solid material with the density as a function of the incident kinetic energy is shown. [From Knoll]
Figure 17-10 The mass attenuation coefficient for energetic photons in lead.

[From Evans]
**Figure 17-11.** Schematic diagrams of (top to bottom) photoelectric effect, Compton effect, and pair production.

**Figure 17-12.** A schematic diagram of Compton scattering.
Figure 17-13 Top: schematic version of Compton scattering. Bottom: schematic variation of the distribution of electron kinetic energies produced in Compton scattering. [From Knoll]

Figure 17-14 A summary of the relative importance of the three mechanisms by which photons interact with matter. The curves indicate the locations in the atomic number-photon energy plane at which the cross section for Compton scattering is equal to that for photoelectric absorption, left side, or is equal to that for pair-production, right side.