12. Nuclear Reactions in Nature: Nuclear Astrophysics

12.1 Introduction

An important mystery that is still unfolding today is how did the chemical elements that we have here on earth come into existence? We know that the readily available chemical elements are restricted in number to less than ninety and that they are essentially immutable by chemical reactions. The large scale nuclear reactions that are taking place are those induced by (external) cosmic rays and radioactive decay, induced nuclear reactions such as fission take place on a tiny scale. Thus, the vast bulk of chemical elements that we have today are those that were present when the earth was formed. The elements have undergone an enormous range of geochemical, geological and biochemical processes but all such processes retain the integrity of the atom. Thus, the origin of the elements is certainly extraterrestrial but questions remain as to where and how they were formed?

These questions lie in the field of Nuclear Astrophysics, an area concerned with the connection of fundamental information on the properties of nuclei and their reactions to the perceived properties of astrological objects and processes that occur in space. The universe is composed of a large variety of massive objects distributed in an enormous volume. Most of the volume is very empty (< 1x10^{-18} kg/m^3) and very cold (~ 3 K). On the other hand, the massive objects, stars and such, are very dense (sun's core ~ 2x10^5 kg/m^3) and very hot (sun's core ~ 16x10^6
These temperatures and densities are such that the light elements are ionized and have high enough thermal velocities to occasionally induce a nuclear reaction. Thus, the general understanding of the synthesis of the heavier elements is that they were created by a variety of nuclear processes in massive stellar systems. These massive objects exert large gravitational forces and so one might expect the new materials to remain in the stars. The stellar processing systems must explode at some point in order to disperse the heavy elements. When we look at the details of the distribution of isotopes here on earth we will find that some number of explosive cycles must have taken place before the earth was formed.

In this chapter we will first consider the underlying information on the elemental abundances and some of the implications of the isotopic abundances. Then we will consider the nuclear processes that took place to produce the primordial elements and those that processed the primordial light elements into those that we have here on earth.

### 12.2 Elemental and Isotopic Abundances

Many students of chemistry have given little thought to the relative abundances of the chemical elements. Everyone realizes that some elements and their compounds are more common than others. The oxygen in water, for example, must be plentiful compared to mercury or gold. But what if we compare elements that are closer in the periodic table, for example, what is the amount of lead (Z=82) compared to mercury (Z=80) or what is the amount iron (Z=26) compared to copper (Z=27)? Oddly enough, the answer one gets depends on what is sampled.
The relative abundances of the first forty elements are shown in Figure 12.1 as a percentage by mass of the earth's crust and as a percentage by mass of our solar system. Notice that the scale is logarithmic and the data spans almost eleven orders of magnitude. The earth is predominantly oxygen, silicon, aluminum, iron and calcium, which comprise more than 90% of the earth's crust. On the other hand, the mass of the solar system is dominated by mass of the sun so that the solar system is mostly hydrogen, with some helium, and everything else is present at the trace level. The differences between the solar system abundances and those on the earth are due to geophysical and geochemical processing of the solar material. Therefore, in this text we will concentrate on understanding the solar system abundances that reflect nuclear processes. The abundances of the isotopes and the elements are the basic factual information that we have to test theories of nucleosynthesis. We have data from the earth, moon, and meteorites, from spectroscopic measurements of the sun, and recently from spectroscopic measurements of distant stars. Many studies have characterized and then attempted to explain the similarities and differences from what we observe in the solar system.
Figure 12.1 The abundances of the first forty elements as a percentage by mass of the earth’s crust (filled squares) and in the solar system (open circles) from the CRC Handbook of Chemistry and Physics, 75th Edition.

The solar abundances of all of the chemical elements are shown in Figure 12.2. These abundances are derived primarily from knowledge of the elemental abundances in CI carbonaceous chondritic meteorites and stellar spectra. Note that ~99% of the mass is in the form of hydrogen and helium. Notice that there is a general logarithmic decline in the elemental abundance with atomic number with the exceptions of a large dip at beryllium (Z=4) and of peaks at carbon and oxygen (Z=6-8), iron (Z ~ 26) and the platinum (Z=78) to lead (Z=82) region. Also notice
that there is a strong odd-even staggering and that all the even Z elements with Z > 6 are more abundant than their odd atomic number neighbors. We have already encountered an explanation for this effect, i.e., recall from earlier discussions on nuclear stability that there are many more stable nuclei for elements with an even number of protons than there are for elements with an odd number of protons simply because there are very few stable odd-odd nuclei. Thus the simple number of stable product nuclei, whatever the production mechanism, will have an effect on the observed populations because nearly all radioactive decay will have taken place since the astrophysical production leaving (only) the stable remains. There are exceptions, of course, and contemporary research emphasizes observing recently produced radioactive nuclei in the cosmos.
Figure 12-2 The abundances of the elements as a percentage by mass of the solar system from the CRC Handbook of Chemistry and Physics, 75th Edition.

Given what we know about nuclear structure, it is reasonable to consider the isotopic distribution rather than the elemental distribution. An example of the isotopic abundances of the top-row elements is shown in Figure 12.3. Once again the very strong staggering is seen and the depression of masses between 5 and 10 is more apparent. This mass region has gaps (no stable nuclei with A = 5 or 8) and the remaining nuclei are all relatively fragile and have small binding energies. For the lightest nuclei, the nuclei whose mass numbers are a multiple of 4 have the highest abundances. Again, simple nuclear stability considerations affect the amount of
beryllium we find relative to the amount of carbon or oxygen, but the many orders of magnitude difference in the abundance of elements like beryllium and carbon must be due to some effects from the production mechanisms.

![Figure 12-3](image)

Figure 12-3 The mass-fractions of the top-row elements in the sun from Anders and Grevesse (1989). Elements with an odd atomic number are shown by square symbols, even by circles. The isotopes of a given chemical element are connected by line segments.

The sun is a typical star (see below) and one uses the solar abundances to represent the elemental abundances in the Universe (the “cosmic” abundances). It will turn out that several nucleosynthetic processes are necessary to explain the details of the observed abundances. In figure 12-4, we jump ahead of our discussion
to show a rough association between the elemental abundances and the nucleosynthetic processes that created them. Figure 12-4 is based upon a pioneering paper by Burbidge, Burbidge, Fowler and Hoyle (B²FH) [23] and an independent analysis of Cameron [24]. These works have served as a framework for the discussion of nucleosynthesis since their publication in the 1950s and we will follow a similar route in our discussion.

![Diagram](image)

Figure 12-4. The atomic abundances of the elements in the solar system and the major nucleosynthetic processes responsible for the observed abundances.

### 12.3 Primordial Nucleosynthesis
The Universe is between 10 and 20 billion years old, with the best estimate of its age being $14 \pm 1 \times 10^9$ years old. The Universe is thought to have begun with a cataclysmic explosion called the Big Bang. Since the Big Bang, the Universe has been expanding with a decrease of temperature and density.

One important piece of evidence to support the idea of the Big Bang is the 2.7 K microwave radiation background in the Universe. This blackbody radiation was discovered by Penzias and Wilson in 1965 and represents the thermal remnants of the electromagnetic radiation that existed shortly after the Big Bang. Weinberg [25] tells how Penzias and Wilson found a microwave noise at 7.35 cm that was independent of direction in their radio antenna at the Bell Telephone Laboratories in New Jersey. After ruling out a number of sources for this noise, they noted a pair of pigeons had been roosting in the antenna. The pigeons were caught, shipped to a new site, reappeared, were caught again and were “discouraged by more decisive means”. The pigeons, it was noted, had coated the antenna with a “white dielectric material”. After removal of this material, the microwave background was still there. It was soon realized that this 7.35 cm radiation corresponded to an equivalent temperature of the noise of about 3.5 K, which was eventually recognized as the remnants of the Big Bang. (Subsequent measurements have characterized this radiation as having a temperature of 2.7 K with a photon density of $\sim 400$ photons/cm$^3$ in the Universe).

A pictorial representation of some of the important events in the “thermal” history of the Universe is shown in Figure 12-5.
Figure 12-5  An outline of the events in the Universe since the Big Bang. From Rolfs and Rodney.

The description of the evolution of the Universe begins at $10^{-43}$ s after the Big Bang, the so-called Planck time. The Universe at that time had a temperature of $10^{32}$ K ($k_B T \sim 10^{19}$ GeV) and a volume that was $\sim 10^{-31}$ of its current volume. [To convert temperature in K to energy ($k_B T$) in electron volts, note that $k_B T (eV) = 8.6 \times 10^{-5} T(K)$] Matter existed in a state unknown to us, a plasma of quarks and gluons. All particles were present and in statistical equilibrium, where each particle had a production rate equal to the rate at which it was destroyed. As the Universe expanded, it cooled and some species fell out of statistical equilibrium. At a time of $10^{-6}$ s ($T \sim 10^{13}$ K), the photons from the black body radiation could not sustain the production of the massive particles and the hadronic matter condensed into a gas of nucleons and mesons. At this point, the Universe consisted of nucleons, mesons, neutrinos (and antineutrinos), photons, electrons (and positrons). The ratio of baryons to photons was $\sim 10^{-9}$. 
At a time of $10^{-2}$ s ($T \sim 10^{11}$ K), the density of the Universe dropped to $\sim 4 \times 10^6$ kg/m$^3$. [In this photon dominated era, the temperature $T$ (K) was given by the relation

$$T(K) = \frac{1.5 \times 10^{10}}{\sqrt{t(s)}}$$

where $t$ is the age in s.] At this time, the neutrons and protons interconvert by the weak interactions

$$\bar{\nu}_e + p \leftrightarrow e^+ + n$$
$$\nu_e + n \leftrightarrow p + e^-$$

[One neglects the free decay of the neutron to the proton because the half-life for that decay (10.6 m) is too long to be relevant.] The neutron-proton ratio, $n/p$, was determined by a Boltzmann factor, i.e.,

$$n/p = \exp(-\Delta mc^2/kT)$$

where $\Delta mc^2$ in the n-p mass difference (1.29 MeV). At $T=10^{12}$ K, $n/p \sim 1$, at $T=10^{11}$ K $n/p \sim 0.86$, etc. At $T = 10^{11}$ K, no complex nuclei were formed because the temperature was too high to allow deuterons to form. When the temperature fell to $T= 10^{10}$ K ($t \sim 1$ s), the creation of $e^+/e^-$ pairs (pair production) ceased because $kT < 1.02$ MeV and the neutron/proton ratio was $\sim 17/83$. At a time of 225 s, this ratio was $13/87$, the temperature was $T \sim 10^9$ K, then density was $\sim 2 \times 10^4$ kg/m$^3$, and the first nucleosynthetic reactions occurred.

These primordial nucleosynthesis reactions began with the production of deuterium by the simple radiative capture process:

$$n + p \rightarrow d + \gamma$$
Notice that the deuteron can be destroyed by the absorption of a high energy photon in the reverse process. At this time, the deuteron survived long enough to allow the subsequent reactions

\[ p + d \rightarrow ^3\text{He} + \gamma \]

and

\[ n + d \rightarrow ^3\text{H} + \gamma \]

$^3\text{H}$ and $^3\text{He}$ are more strongly bound allowing further reactions that produce the very strongly bound alpha particles

\[ ^3\text{H} + p \rightarrow ^4\text{He} + \gamma \]

\[ ^3\text{He} + n \rightarrow ^4\text{He} + \gamma \]

\[ ^3\text{H} + d \rightarrow ^4\text{He} + n \]

\[ d + d \rightarrow ^4\text{He} + \gamma \]

Further reactions to produce the $A=5$ nuclei do not occur because there are no stable nuclei with $A=5$ (or $A=8$). A small amount of $^7\text{Li}$ is produced in the reactions

\[ ^4\text{He} + ^3\text{H} \rightarrow ^7\text{Li} + \gamma \]

\[ ^3\text{He} + ^3\text{He} \rightarrow ^7\text{Be} + \gamma \]

but the $^7\text{Li}$ is also very weakly bound and is rapidly destroyed. Thus, the synthesis of larger nuclei was blocked. After about 30 m, nucleosynthesis ceased. The temperature was \( \sim 3 \times 10^8 \text{K} \) and the density was \( \sim 30 \text{ kg/m}^3 \). (For reference, please note that water vapor at 1 atm has a density of \( \sim 1 \text{ kg/m}^3 \) and liquid water has a density of \( \sim 10^3 \text{ kg/m}^3 \)). Nuclear matter was 76% by mass protons, 24% alpha particles with traces of deuterium, $^3\text{He}$ and $^7\text{Li}$. The $\gamma/n/p$ ratio is $10^9/13/87$. The relative ratio of $p/^4\text{He}/d/^3\text{He}/^7\text{Li}$ is a sensitive function of the baryon density of the
Universe (Figure 12.6), a fact that is used to constrain models of the Big Bang. (The cross sections for the reactions that convert one product to another are generally known and complex network calculations of the reaction rates can be performed as a function of temperature and density. The resulting abundances can be compared to estimates from observations of stellar matter.)
Figure 12-6 The variation of the relative abundances of the Big-Bang nuclei and the $^4\text{He}$ mass fraction versus the baryon density. The boxes indicate the measured data and an estimate of the uncertainty. The curves indicate the dependence of the yield on the baryon density in the Big Bang; the vertical bar indicates the region of overlap.

Chemistry began about $10^6$ years later, when the temperature has fallen to 2000K and the electrons and protons could combine to form atoms. Further nucleosynthesis continues to occur in the interiors of stars.

12.4 Stellar Evolution

As discussed above, nucleosynthesis occurred in two steps, the primordial nucleosynthesis that occurred in the Big Bang forming only the lightest nuclei and
later processes, beginning ~ 10^6 years after the Big Bang and continuing to the present, of nucleosynthesis in the stars. Big Bang nucleosynthesis produced hydrogen, helium and traces of \(^{7}\)Li while the rest of the elements are the result of stellar nucleosynthesis. For example, recent observations of stellar spectral lines showing the presence of \(2 \times 10^5\) y \(^{99}\)Tc indicate on-going stellar nucleosynthesis. To understand the nuclear reactions that make the stars shine and generate the bulk of the elements, one needs to understand how stars work. That is the focus of this section.

After the Big Bang explosion, the material of the Universe was dispersed. Inhomogeneities developed, which under the influence of gravity, condensed to form the galaxies. Within these galaxies, clouds of hydrogen and helium gas can collapse under the influence of gravity. At first, the internal heat of this collapse is radiated away. As the gas becomes denser, however, the opacity increases, and the gravitational energy associated with the collapse is stored in the interior rather than being radiated into space. Eventually a radiative equilibrium is established with the development of a protostar. The protostar continues to shrink under the influence of gravity with continued heating of the stellar interior. When the interior temperature reaches ~10^7 K, thermonuclear reactions between the hydrogen nuclei can begin because some of the protons have sufficient kinetic energies to overcome the Coulomb repulsion between the nuclei.

The first generation of stars that formed in this way is called population III stars. They consisted of hydrogen and helium, were massive, had relatively short
lifetimes and are now extinct. The debris from these stars has been dispersed and was incorporated into later generation stars.

The second generation of stars, called *population II stars*, was comprised of hydrogen, helium, and about 1% of the heavier elements like carbon and oxygen. Finally there was a third generation of stars, like our sun, called *population I stars*. These stars consist of hydrogen, helium, and 2.5% of the heavier elements.

Our sun is a typical population I star. It has a mass of $2.0 \times 10^{30}$ kg, a radius of $7.0 \times 10^6$ m, (an average density of $1.41 \times 10^3$ kg/m$^3$), a surface temperature of $\sim 6000$ K and a luminosity of $3.83 \times 10^{26}$W. Our sun is $4.5 \times 10^9$ years old.

The Danish astronomer Ejnar Hertzsprung and the American astronomer Henry Norris Russell independently observed a very well-defined correlation between the luminosity and surface temperature of stars. That correlation is shown in Figure 12.7 and is called a Hertzsprung-Russell or H-R diagram. Most stars, like our sun, fall in a narrow band on this diagram called the *main sequence*. Stars in this main sequence have luminosities, $L$, that are approximately proportional to $T_{\text{surface}}^{5.5}$, or in terms of their mass, $M$, $L \propto M^{3.5}$. How long a star stays on the main sequence will depend on its mass, which, in turn, is related to the reaction rates in its interior.

In the upper right portion of the H-R diagram, one sees a group of stars, the *red giants or super giants*, with large radii that are relatively cool (3000-4000 K). Stars on the main sequence move to this region when the nuclear energy liberated in the nuclear reactions occurring in the star is not enough to sustain main sequence luminosity values.
Figure 12-7 A schematic representation of a Hertzsprung-Russell diagram. From Rolfs and Rodney.

Our sun is expected to spend $\sim 7 \times 10^9$ more years on the main sequence before becoming a red giant. In a shorter time of $1.1 - 1.5 \times 10^9$ years, the sun will increase slowly in luminosity by $\sim 10\%$, probably leading to a cessation of life on earth. (In short, terrestrial life has used up $\sim \frac{3}{4}$ of its allotted time, since it began $\sim 3.5 \times 10^9$ years ago.)

In the lower left of the H-R diagram, one sees a group of small dense, bright stars ($T > 10^4$ K) called white dwarfs. The white dwarfs represent one evolutionary outcome for the red giants with masses between 0.1 and 1.4 solar masses. Red giants are helium-burning stars and as the helium is burnt, the stars become unstable, ejecting their envelopes, creating a planetary nebula and moving across the main sequence on the H-R diagram to be white dwarfs. (See Figure 12.7 for a schematic view of this evolution).
For massive red giants (M > 8 solar masses), one finds they undergo a more spectacular death cycle, with contractions, increases in temperature leading to helium burning, carbon-oxygen burning, silicon burning, etc. with the production of the elements near iron, followed by an explosive end (Figure 12.8).

Figure 12.8 Schematic diagram of the evolution of (a) a star with a mass near that of the sun and (b) a more massive star. From Rolfs and Rodney.

The explosive end for these stars can lead to the formation of novae and supernovae. The name “nova” means “new” and connotates a star that undergoes a sudden increase in brightness, followed by a fading characteristic of an explosion. In this process, the outer part of the star containing only ~ 10^{-3} of the stellar mass, is ejected with the release of ~ 10^{45} ergs. Supernovae are spectacular stellar explosions in which the stellar brightness increases by a factor of 10^6 – 10^9,
releasing $\sim 10^{51}$ ergs on a time scale of seconds. We have observed about 10 nova/year but only 2-3 supernova per century. Supernovae are classified as Type I (low hydrogen, high “heavy” elements, such as oxygen to iron), and Type II (primarily hydrogen, with lesser amounts of the “heavy” elements). Some supernovae lead to the formation of neutron stars, which are giant nuclei of essentially pure neutronic matter.

12.5 Thermonuclear Reaction Rates

Before discussing the nuclear reactions involved in stellar nucleosynthesis, we need to discuss the rates of these reactions which take place in a “thermal soup” as opposed to reactions studied in the laboratory. The rates of these reactions will tell us what reactions are most important. When we speak of thermonuclear reactions, we mean nuclear reactions in which the energy of the colliding nuclei is the thermal energy of the particles in a hot gas. Both reacting nuclei are moving and thus it is their relative velocity (center of mass energy) that is important. In ordinary nuclear reactions in the laboratory, we write for the rate of the reaction, $R$,

$$R = N \sigma \phi$$

where the reaction rate $R$ is in reactions/s, $\sigma$ is the reaction cross section (in cm$^2$), $\phi$ the incident particle flux in particles/s and $N$ is the number of target atoms/cm$^2$.

For astrophysical reactions, we write

$$R = N_x N_y \int_0^\infty \sigma(v)v \, dv = N_x N_y \langle v \sigma \rangle$$

where $v$ is the relative velocity between nuclei $x$ and $y$, each present in a concentration of $N_i$ particles/cm$^3$, the quantity $\langle \sigma v \rangle$ is the temperature averaged
reaction rate per particle pair. (To be sure that double counting of collisions between identical particles does not occur, it is conventional to express the above equation as

\[ R = \frac{N_x N_y \langle \alpha \nu \rangle}{1 + \delta_{xy}} \]

where \( \delta_{xy} \) is the Kronecker delta (which is 0 when \( x \neq y \) and 1 when \( x = y \)). Note the mean lifetime of nuclei \( x \) is then \( 1/(N_x \langle \alpha \nu \rangle) \).

In a hot gas the velocity distribution of each component will be given by a Maxwell-Boltzmann distribution

\[ P(\nu) = \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left( -\frac{m\nu^2}{2kT} \right) 4\pi \nu^2 d\nu \]

where \( m \) is the particle mass, \( k \) is Boltzmann’s constant and \( T \) is the gas temperature. Integrating over all velocities for the reacting particles, \( x \) and \( y \), gives

\[ \langle \alpha \nu \rangle = \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{1/2}} \int_0^\infty \sigma(E)E \exp\left(-\frac{E}{kT}\right) dE \]

where \( \mu \) is the reduced mass \( \left( \frac{m_x m_y}{m_x + m_y} \right) \). The rates, \( R \), of stellar nuclear reactions are directly proportional to \( \langle \alpha \nu \rangle \), which depends on the temperature \( T \).

For slow neutron induced reactions that do not involve resonances, we know (Chapter 10) that \( \sigma_n(E) \propto \frac{1}{\nu_n} \) so that \( \langle \alpha \nu \rangle \) is a constant. For charged particle reactions, one must overcome the repulsive Coulomb force between the positively charged nuclei. For the simplest reaction, \( p + p \), the Coulomb barrier is 550 keV. But, in a typical star like the sun, \( kT \) is 1.3 keV, i.e., the nuclear reactions that occur
are sub-barrier and the resulting reactions are the result of barrier penetration. (At a proton-proton center of mass energy of 1 keV, the barrier penetration probability is \( \sim 2 \times 10^{-10} \)). At these extreme sub-barrier energies, the barrier penetration factor can be approximated as

\[
P = \exp\left(-\frac{2\pi Z_1 Z_2 e^2}{h v}\right) = \exp\left(-31.29 Z_1 Z_2 \left(\frac{\mu}{E}\right)^{1/2}\right)
\]

where \( E \) is in keV and \( \mu \) in amu. This tunneling probability is referred to as the Gamow factor. The cross section (Chapter 10) is also proportional to \( \pi \lambda^2 \propto \frac{1}{E} \).

Thus the cross section for non-resonant charged particle induced reactions can be written as

\[
\sigma(E) = \frac{1}{E} \exp\left(-31.29 Z_1 Z_2 \left(\frac{\mu}{E}\right)^{1/2}\right) S(E)
\]

where the function \( S(E) \), the so-called astrophysical \( S \) factor, contains all the constants and terms related to the nuclei involved. Substituting this expression into the equation for \( \langle \alpha v \rangle \), we have

\[
\langle \alpha v \rangle = \left(\frac{8}{\pi \mu}\right)^{1/2} \frac{1}{(kT)^{1/2}} \int_0^\infty S(E) \exp\left[-\frac{E}{kT} - \frac{b}{E^{1/2}}\right] dE
\]

where \( b \) is \( 0.989 Z_1 Z_2 \mu^{1/2} \text{(MeV)}^{1/2} \). This equation represents the overlap between the Maxwell-Boltzmann distribution which is peaked at low energies and the Gamow factor which increases with increasing energy. The product of these two terms produces a peak in the overlap region of these two functions called the Gamow peak (Figure 12.9). This peak occurs at an energy \( E_0 = (b kT/2)^{2/3} \).
Figure 12.9 Rate of non-resonant stellar nuclear reactions as a function of temperature. From Wong

For reactions involving isolated single resonances or broad resonances, it is possible to derive additional formulae for $\sigma(E)$ [R+R] in the Breit-Wigner form, i.e.,

$$
\sigma(E) = \pi \lambda^2 \left[ \frac{2J_x + 1}{(2J_x + 1)(2J_y + 1)} \right] \frac{\Gamma_{in} \Gamma_{out}}{(E - E_r)^2 + \frac{\Gamma_{tot}^2}{4}}
$$

where $J_x, J_y, J_r$ are the spins of the interacting particles and the resonance and $\Gamma_{in}$, $\Gamma_{out}, \Gamma_{tot}$ are the partial widths of the entrance and exit channels and the total width, respectively.

12.6 Stellar Nucleosynthesis

12.6.1 Introduction

After Big Bang nucleosynthesis, we have a universe that is $\sim 75\%$ hydrogen and $\sim 25\%$ helium with a trace of $^7$Li. Stellar nucleosynthesis continues the synthesis of the chemical elements. Beginning $\sim 10^6$ years after the Big Bang, as described in Section 12.4, the sequence of gravitational collapse, and an increasing temperature causes the onset of nuclear fusion reactions, releasing energy that stops the collapse. Starting from hydrogen and helium, fusion reactions produce the nuclei up to the maximum in the nuclear binding energy curve at $A \sim 60$. The
limiting temperature of these reactions is about $5 \times 10^9$ K, where $kT \sim 0.4$ MeV. A rough outline of the nuclear reactions involved is given in Table 12.1

<table>
<thead>
<tr>
<th>Fuel</th>
<th>$T(K)$</th>
<th>$kT(\text{MeV})$</th>
<th>Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1\text{H}$</td>
<td>$5 \times 10^7$</td>
<td>0.002</td>
<td>$^4\text{He}$</td>
</tr>
<tr>
<td>$^4\text{He}$</td>
<td>$2 \times 10^8$</td>
<td>0.02</td>
<td>$^{12}\text{C}$, $^{16}\text{O}$, $^{20}\text{Ne}$</td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>$8 \times 10^8$</td>
<td>0.07</td>
<td>$^{16}\text{O}$, $^{20}\text{Ne}$, $^{24}\text{Mg}$</td>
</tr>
<tr>
<td>$^{16}\text{O}$</td>
<td>$2 \times 10^9$</td>
<td>0.2</td>
<td>$^{20}\text{Ne}$, $^{28}\text{Si}$, $^{32}\text{S}$</td>
</tr>
<tr>
<td>$^{20}\text{Ne}$</td>
<td>$1.5 \times 10^9$</td>
<td>0.13</td>
<td>$^{16}\text{O}$, $^{24}\text{Mg}$</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>$3.5 \times 10^9$</td>
<td>0.3</td>
<td>$A &lt; 60$</td>
</tr>
</tbody>
</table>

The products from these reactions are distributed into the galaxies by slow emission from the red giants and by the catastrophic explosions of novae and supernovae. This dispersed material condenses in the Population II and later the population I stars where additional nuclear reactions (see below) create the odd A nuclei and sources of free neutrons. These neutrons allow us to get slow neutron capture reactions (s process) synthesizing many of the nuclei with $A > 60$. High temperature photonuclear reactions and rapid neutron capture reactions in supernovae complete the bulk of the nucleosynthesis reactions.

12.6.2 Hydrogen Burning

The first stage of stellar nucleosynthesis, which is still occurring in stars like our sun, is *hydrogen burning*. In hydrogen burning, protons are converted to $^4\text{He}$ nuclei. Since there are no free neutrons present, the reactions differ from those of Big Bang nucleosynthesis. The first reaction that occurs is
\[
p + p \rightarrow d + e^+ + \nu_e \quad Q = 0.42 \text{ MeV}
\]
where most of the released energy is shared between the two leptons. In our sun, \( T \sim 15 \times 10^6 \text{ K} \) (\( kT \sim 1 \text{ keV} \)). The proton-proton (pp) reaction is a weak interaction process and has, therefore, a very small cross section, \( \sim 10^{-47} \text{ cm}^2 \), at these proton energies. The resulting reaction rate is \( 5 \times 10^{-18} \text{ reactions/s/proton} \).

There is an improbable (0.4\%) variant of this reaction, called the pep reaction that also leads to deuteron production. It is

\[
p + e^- + p \rightarrow d + \nu_e \quad Q = 1.42 \text{ MeV}
\]
This rare reaction is a source of energetic neutrinos from the sun.

The next reaction in the sequence is

\[
d + p \rightarrow ^3\text{He} + \gamma \quad Q = 5.49 \text{ MeV}.
\]
leading to the synthesis of \(^3\text{He} \). The rate of this strong interaction is \( \sim 10^{16} \) times greater than the weak \( p + p \) reaction. The product \(^3\text{He} \) can undergo two possible reactions. In \( \sim 86\% \) of the cases, the reaction is

\[
^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2p \quad Q = 12.96 \text{ MeV}
\]
This reaction, when combined with the two previous reactions (\( p + p \) and \( d + p \)) corresponds to an overall reaction of

\[
4p \rightarrow ^4\text{He} + 2e^+ + 2\nu_e \quad Q = 26.7 \text{ MeV}
\]
This sequence of reactions is called the ppI chain and is responsible for 91\% of the sun’s energy. A schematic view of this reaction is shown in Figure 12.10.
In ~ 14% of the cases, the $^3$He product undergoes the side reaction with an alpha particle

\[ ^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \nu_e \]

The $^7$Be undergoes subsequent electron capture decay

\[ e^- + ^7\text{Be} \rightarrow ^7\text{Li} + \nu_e \quad Q = 0.86 \text{ MeV} \]

Note that this EC decay process does not involve capture of the orbital electron of the $^7$Be since it is fully ionized in a star, but rather involves capture of a free continuum electron. As a consequence, the half-life of this decay is ~ 120 d rather than the terrestrial half-life of 77 days. The resulting $^7$Li undergoes proton capture as

\[ p + ^7\text{Li} \rightarrow 2 \ ^4\text{He} \]

This sequence of reactions ($p+p$, $d+p$, $^3\text{He} + ^4\text{He}$, $^7\text{Be}$ EC, $^7\text{Li} (p,\alpha)$ constitutes the ppII process which accounts for ~ 7% of the sun’s energy.
A small fraction of the $^7\text{Be}$ from the $^3\text{He} + ^4\text{He}$ reaction can undergo proton capture, leading to

$$^7\text{Be} + p \rightarrow ^8\text{B} + \gamma$$

$$^8\text{B} \rightarrow ^8\text{Be}^* + e^* + \nu_e$$

$$^8\text{Be}^* \rightarrow 2^4\text{He}$$

This sequence ($p+p$, $p+d$, $^3\text{He} + ^4\text{He}$, $^7\text{Be}(p,\gamma)$, $^8\text{B} \rightarrow ^8\text{Be}^* \rightarrow 2^4\text{He}$) constitutes the ppIII chain (which provides about 0.015% of the sun's energy). In each of the pp processes, some energy is carried away by the emitted neutrinos. Quantitatively, in the ppI process, the loss is 2%, in the ppII process 4%, and 28.3% in the ppIII process. These pp chains are shown together in Figure 12.11

![Diagram of the three pp chains](image)

Figure 12-4 The three chains of nuclear reactions that constitute hydrogen burning and convert protons into $^4\text{He}$. The rate-limiting step in all reactions is the first reaction to create the deuterium.
In population II and population I stars, heavy elements like carbon, nitrogen and oxygen are present, leading to the occurrence of another set of nuclear reactions whose net effect in the conversion $4p \rightarrow ^4\text{He} + 2e^+ + 2\nu_e$. The “heavy” nuclei act as catalysts for this reaction. A typical cycle is

$$^{12}\text{C} + p \rightarrow ^{13}\text{N} + \gamma$$

$$^{13}\text{N} \rightarrow ^{13}\text{C} + e^+ + \nu_e$$

$$^{13}\text{C} + p \rightarrow ^{14}\text{N} + \gamma$$

$$^{14}\text{N} + p \rightarrow ^{15}\text{O} + \gamma$$

$$^{15}\text{O} \rightarrow ^{15}\text{N} + e^+ + \nu_e$$

$$^{15}\text{N} + p \rightarrow ^{12}\text{C} + ^4\text{He}$$

This group of reactions is referred to as the CNO cycle, and is favored at higher temperatures where the Coulomb barrier for these reactions can be more easily overcome. In our sun, 98% of the energy comes from the pp chain and only 2% from the CNO cycle. Several side chains of this reaction cycle are possible, as illustrated in Figure 12.12.
12.6.3 Helium Burning

Eventually the hydrogen fuel of the star will be exhausted and further gravitational collapse will occur. This will give rise to a temperature increase up to $\sim 1 - 2 \times 10^8$ K (with a density of $\sim 10^8$ kg/m$^3$). In this red giant, helium burning will commence.

On might think the first reaction might be

$$^4\text{He} + ^4\text{He} \rightarrow ^8\text{Be} \quad Q = -0.0191 \text{ MeV}$$

but $^8\text{Be}$ is unstable ($t_{1/2} = 6.7 \times 10^{-17}$ s) and thus that process is hindered by the short lifetime and low transient population of the Be nuclei. Instead one gets the so-called “3α” process

$$3 ~ ^4\text{He} \rightarrow ^{12}\text{C} \quad Q = 7.37 \text{ MeV}$$

Three body reactions are rare but the reaction proceeds through a resonance in $^{12}\text{C}$ at 7.65 MeV corresponding to the second excited state of $^{12}\text{C}$ ($J\pi =0^+$). This excited state has a more favorable configuration than the $^{12}\text{C}$ ground state for allowing the collision to occur. (In a triumph for nuclear astrophysics, the existence of this state was postulated by astrophysicists to explain nucleosynthetic rates before it was found in the laboratory.)

After a significant amount of $^{12}\text{C}$ is formed, one gets the alpha capture reactions

$$^4\text{He} + ^{12}\text{C} \rightarrow ^{16}\text{O} + \gamma \quad Q = 7.16 \text{ MeV}$$

$$^4\text{He} + ^{16}\text{O} \rightarrow ^{20}\text{Ne} + \gamma \quad Q = 4.73 \text{ MeV}$$
A brief interlude of *neon burning* then occurs with reactions like

\[ ^{20}\text{Ne} + \gamma \rightarrow ^{4}\text{He} + ^{16}\text{O} \]

\[ ^{4}\text{He} + ^{20}\text{Ne} \rightarrow ^{24}\text{Mg} + \gamma \]

The relative rates of these and related processes are shown in Figure 12.13.

![Figure 12.13](image)

Figure 12.13 Mean lifetimes for various nucleosynthesis reactions involving alpha particles as a function of temperature. The mean lifetime is inversely related to the reaction rate. From B²FH.

**12.6.4 Synthesis of Nuclei with A < 60**

Eventually the helium of the star will be exhausted, leading to further gravitational collapse with a temperature increase to \(6 \times 10^8 \sim 2 \times 10^9\)K (\(kT \sim 100-200\) keV). At this point the fusion reactions of the “alpha cluster” nuclei are possible. For example, carbon and oxygen burning occurs in charged particle reactions such as

\[ ^{12}\text{C} + ^{12}\text{C} \rightarrow ^{20}\text{Ne} + ^{4}\text{He} \]
with the production of $^{28}\text{Si}$ and $^{32}\text{S}$ being the most important branches of the oxygen-burning reactions. Further rises in temperature up to $5 \times 10^9$ K result in a series of silicon burning reactions involving an equilibrium between photodisintegration and radiative capture processes such as

$$^{28}\text{Si} + \gamma \rightarrow ^{24}\text{Mg} + ^4\text{He}$$

$$^4\text{He} + ^{28}\text{Si} \rightarrow ^{32}\text{S} + \gamma$$

The nuclei up to $A \sim 60$ are produced in these equilibrium processes. In such equilibrium processes, the final yields of various nuclei are directly related to their nuclear stability with the more stable nuclei having higher yields. So one observes greater yields of e-e nuclei than odd $A$ nuclei (due to the pairing term in the mass formula) and even $N$ isotopes are more abundant than odd $A$ isotopes of an element.

The relative time scales of the various reactions leading to nuclei with $A < 60$ are shown in Table 12.2. Note these time scales are inversely proportional to the reaction rates.

Table 12.2 Time Scales of Nucleosynthetic Reactions in a 1 Solar Mass Star
<table>
<thead>
<tr>
<th>Reaction</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>H burning</td>
<td>$6 \times 10^9$ years</td>
</tr>
<tr>
<td>He burning</td>
<td>$0.5 \times 10^6$ years</td>
</tr>
<tr>
<td>C burning</td>
<td>200 years</td>
</tr>
<tr>
<td>Ne burning</td>
<td>1 year</td>
</tr>
<tr>
<td>O burning</td>
<td>Few months</td>
</tr>
<tr>
<td>Si burning</td>
<td>days</td>
</tr>
</tbody>
</table>

12.6.5 Synthesis of nuclei with $A > 60$

The binding energy per nucleon curve peaks at $A \sim 60$ and decreases as $A$ increases beyond 60. This indicates that fusion reactions using charged particles are not energetically favored to make these nuclei. However, another possible nuclear reaction is neutron capture, i.e., $(n,\gamma)$. These reactions have no Coulomb barriers to inhibit them and the rates are then governed by the Maxwell-Boltzmann distribution of velocities in a hot gas and the availability of free neutrons. If $\sigma(n,\gamma) \sim 1/v$, then the reaction rate $N_n<\sigma v>$ is largely governed by $N_n$, the neutron density. We can identify two types of neutron capture processes for nucleosynthesis. The first of these is *slow neutron capture, the so-called s process*, where the time scale of the neutron capture process, $\tau_{\text{reaction}} \gg \tau_\beta$, where $\tau_\beta$ is the $\beta^-$ decay lifetime. In this process, each neutron capture proceeds in competition with $\beta^-$ decay. For example, consider the stable nucleus $^{56}\text{Fe}$. If it captures a neutron the following reactions can occur

$$^{56}\text{Fe} + n \rightarrow ^{57}\text{Fe} \text{ (stable)} + \gamma$$
$^{57}\text{Fe} + n \rightarrow ^{58}\text{Fe} \text{ (stable)} + \gamma$

$^{58}\text{Fe} + n \rightarrow ^{59}\text{Fe} \left( t_{1/2} = 44.5 \text{ d} \right) + \gamma$

Then 44.5 d $^{59}\text{Fe}$ will undergo $\beta^-$ decay before another neutron is captured, i.e.,

$^{59}\text{Fe} \rightarrow ^{59}\text{Co} \text{ (stable)} + \beta^- + \bar{\nu}_e$

and further captures will start with $^{59}\text{Co}$. The mean times of neutron capture reactions $\tau_{\text{reaction}} = \ln 2 / \text{rate} = \ln 2 / N_n \langle \sigma v \rangle$. If $N_n \sim 10^{11}/m^3$, $\sigma = 0.1 \text{ b}$, $E_n \sim 50 \text{ keV}$, then $\tau \sim 10^5 \text{ years}$. Then one will see neutron capture by all stable nuclei and many of the long-lived nuclei. A typical s-process path of nucleosynthesis for the nuclei with $Z = 45 - 60$ is shown in Figure 12.14. The production of nuclei follows a zig-zag path through the chart of nuclides, with increases in mass when a neutron is captured and increases in atomic number when $\beta$-decay precedes the next neutron capture.
Figure 12.14 A section of the chart of nuclides showing the s-process path. From Rolfs and Rodney.

The s-process terminates at $^{209}$Bi because of the cyclic sequence

$$^{209}\text{Bi}(n,\gamma)^{210}\text{Bi} \rightarrow ^{210}\text{Po} \rightarrow ^{206}\text{Pb} \rightarrow ^{209}\text{Pb} \rightarrow ^{209}\text{Bi}$$

The source of the neutrons for the s-process is $(\alpha, n)$ reactions on n-rich nuclei like $^{13}$C or $^{21}$Ne, with the latter being most important. In population II and I stars, one can get side reactions like

$$^{20}\text{Ne}(p,\gamma)^{21}\text{Na}$$

$$^{21}\text{Na} \rightarrow ^{21}\text{Ne} + e^+ + \nu_e$$

that produce the target nuclei for the $(\alpha, n)$ reactions.

For the slow neutron capture process, there is an equilibrium between the production and loss of adjacent nuclei. Stable nuclei are only destroyed by neutron capture. For such nuclei, we can write for the rate of change of a nucleus with mass number $A$

$$\frac{dN_A}{dt} = \sigma_{A-1}N_{A-1} - \sigma_A N_A$$

where $\sigma_i$ and $N_i$ are the capture cross sections and number of nuclei (abundance) for nucleus $i$, respectively. At equilibrium,

$$\frac{dN_A}{dt} = 0$$

Thus

$$\sigma_{A-1}N_{A-1} = \sigma_A N_A$$

This relationship between the abundances of neighboring stable nuclei is a signature for the s-process.
If the time scale of neutron capture reactions is very much less than β-decay lifetimes, then *rapid neutron capture* or the *r-process* occurs. For r-process nucleosynthesis, one needs large neutron densities, $\sim 10^{28} \text{/m}^3$ which lead to capture times of the order of fractions of a second. The astrophysical environment where such processes can occur is now thought to be in supernovae. In the r-process, a large number of sequential captures will occur until the process is terminated by neutron emission or, in the case of the heavy elements, fission or β-delayed fission. The lighter “seed” nuclei capture neutrons until they reach the point where β-decay lifetimes have decreased and β-decay will compete with neutron capture. The r-process is responsible for the synthesis of all nuclei with $A > 209$ and many lower mass nuclei. In a plot of abundances vs mass number $A$ (Figure 12.4) one sees two peaks in the abundance distributions near the magic neutron numbers ($N = 50, 82, 126$). The lower $A$ peak is due to the r-process, which reaches the magic number of neutrons at a lower $Z$ value than the s-process. The peaks occur because of the relative stability of $N=50, 82, 126$ nuclei against neutron capture. A typical r-process path is shown in figure 12.15. Notice that the path climbs up in atomic number along the neutron magic numbers. The nuclei in each zig-zag region are the places of maxima in the isotopic yields after decay.
Another important process leading to the synthesis of some proton rich nuclei with $70 < A < 200$ is the so-called $p$-process. The $p$-process consists of a series of photonuclear reactions $(\gamma, p), (\gamma, \alpha), (\gamma, n)$ on “seed” nuclei from the s or r-processes that produce these nuclei. (Originally it was believed that proton capture processes during supernovae were responsible for these nuclei but it was found that the proton densities are too small to explain the observed abundances). The temperature during a supernovae explosion is $\sim 3 \times 10^9$ K, producing blackbody radiation that can cause photonuclear reactions. The $p$-process contribution to the abundances of most elements is very small but there are some nuclei ($^{190}$Pt, $^{168}$Yb) that seem to have been exclusively made by this process. The relative importance of s, r, and p processes in nucleosynthesis in a given region is shown in Figure 12.16.
Figure 12.16 Typical portion of the heavy element chart of the nuclides showing the relative importance of s, r and p processes in nucleosynthesis. From Truran.

A process that is sometimes related to the p-process is the rp process, the rapid proton capture process. This process makes proton-rich nuclei with $Z = 7$-26.

This process involves a set of $(p, \gamma)$ and $\beta^+$ decays that populate the p-rich nuclei. The process starts as a “breakout” from the CNO cycle, i.e., a side chain of the CNO cycle that produces p-rich nuclei like $^{21}$Na and $^{19}$Ne. These “seed” nuclei can form the basis for further proton captures, leading to the nucleosynthetic path shown in Figure 12.17. The rp process creates a small number of nuclei with $A < 100$. The process follows a path analogous to the r process but on the proton rich side of stability. At present, the source of the protons for this process are certain binary stars.
Figure 12.17 The position of the rp process relative to the line of beta stability.

Note this process while close to the line of beta stability, approaches the proton drip line as the nuclei become heavier.

12.7 Solar Neutrino Problem

12.7.1 Introduction

Many of the nuclear reactions that provide the energy of the stars also result in the emission of neutrinos. Because of the small absorption cross sections for neutrinos interacting with matter ($\sigma_{\text{abs}} \sim 10^{-44} \text{ cm}^2$), these neutrinos are not absorbed in the sun and other stars. (This loss of neutrinos corresponds to a loss of $\sim 2\%$ of the energy of our sun). Because of this, the neutrinos are a window into the stellar interior. The small absorption cross sections also make neutrinos difficult to
detect, with almost all neutrinos passing through the planet earth without interacting.

Recently, a good deal of attention has been given to the “solar neutrino problem” and its solution. The 2002 Nobel Prize in physics was awarded to Ray Davis and Masatoshi Koshita for their pioneering work on this problem. Of special interest is the important role of nuclear and radiochemistry in this work as Davis is a nuclear chemist. The definition and solution of this problem is thought to be one of the major scientific advances of recent years.

12.7.2 Expected Solar Neutrino Sources, Energies and Fluxes

The sun is major source of neutrinos reaching the surface of the earth due to its close proximity. The sun emits $1.8 \times 10^{38}$ neutrinos/s which, after an $8$ minute transport time, reach the surface of the earth at a rate of $6.4 \times 10^{10}$ neutrinos/s/cm$^2$. The predictions of the standard solar model for the neutrino fluxes at the surface of the earth due to various nuclear reactions are shown in Table 12.3

<table>
<thead>
<tr>
<th>Source</th>
<th>Flux (particles/s/cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pp$</td>
<td>$5.94 \times 10^{10}$</td>
</tr>
<tr>
<td>$pep$</td>
<td>$1.40 \times 10^{8}$</td>
</tr>
<tr>
<td>$hep$</td>
<td>$7.88 \times 10^{3}$</td>
</tr>
<tr>
<td>$^7Be$</td>
<td>$4.86 \times 10^{7}$</td>
</tr>
</tbody>
</table>

Table 12.3 Predicted solar neutrino fluxes (Bahcall and Pena-Garay)
The predicted energy distributions are shown in Figure 12.18. Each nuclear reaction has a characteristic neutrino energy distribution.

Figure 12.18 Log-log plot of predicted neutrino fluxes from various solar nuclear reactions. The energy regions to which the neutrino detectors are sensitive are shown at the top. From Bahcall
The source labeled “pp” in the Table 12.3 and Figure 12-18 refers to the reaction

\[ p + p \rightarrow d + e^+ + \nu_e \]

and is the most important reaction, producing one neutrino for each \(^4\)He nucleus made. The “pep” source is the reaction

\[ p + p + e^- \rightarrow d + \nu_e \]

which produces monoenergetic neutrinos, while the source “hep” refers to the reaction

\[ p + ^3\text{He} \rightarrow ^4\text{He} + e^+ + \nu_e \]

This latter reaction produces the highest energy neutrinos with a maximum energy of 18.77 MeV due to the high reaction Q value. The intensity of this source is about \(10^7\) less than the pp source. The “\(^7\)Be” source refers to the pp chain electron capture decay reaction

\[ e^- + ^7\text{Be} \rightarrow ^7\text{Li} + \nu_e \]

This reaction produces two groups of neutrinos, one in which the ground state of \(^7\)Li is populated (90% abundance) and one in which the 0.477 MeV excited state is populated (10% abundance). The source “\(^8\)B” refers to the positron decay

\[ ^8\text{B} \rightarrow ^8\text{Be}^* + e^+ + \nu_e \]

in which the first excited state of \(^8\)Be (at 3.040 MeV) is populated. The weak sources “\(^{13}\)N”, “\(^{15}\)O” and “\(^{17}\)F” refer to \(\beta^+\) decays that occur in the CNO cycle, \(i.e.,\)

\[ ^{13}\text{N} \rightarrow ^{13}\text{C} + \beta^+ + \nu_e \]

\[ ^{15}\text{O} \rightarrow ^{15}\text{N} + \beta^+ + \nu_e \]

\[ ^{17}\text{F} \rightarrow ^{17}\text{O} + \beta^+ + \nu_e \]
12.7.3 Detection of neutrinos

As indicated above, the detection of these weakly interacting neutrinos is difficult because of the low absorption cross sections. Two classes of detectors have been used to overcome this obstacle, radiochemical detectors and Cerenkov detectors. Radiochemical detectors rely on detecting the products of neutrino-induced nuclear reactions while the Cerenkov detectors observe the scattering of neutrinos. The most famous radiochemical detector is that constructed by Davis and co-workers in the Homestake Gold Mine in South Dakota. In a cavern about 1500 m below the surface of the earth, a massive detector, consisting of 100,000 gallons of a cleaning fluid, $\text{C}_2\text{Cl}_4$, was mounted. The cleaning fluid weighed 610 tons and corresponded to the volume of 10 railway tanker cars. The nuclear reaction occurring in the detector was

$$\nu_e + ^{37}\text{Cl} \rightarrow ^{37}\text{Ar} + e^-$$

The $^{37}\text{Ar}$ product nucleus decays by electron capture with a 35-day half-life. After the cleaning fluid has been exposed to solar neutrinos for a period of time, the individual $^{37}\text{Ar}$ product nuclei are flushed from the detector with a stream of He gas and put into a proportional counter where the 2.8 keV Auger electrons from the EC decay are detected. The detection reaction has a threshold of 0.813 MeV making it sensitive to the $^8\text{B}$, hep, pep, and $^7\text{Be}$ (ground state decay) neutrinos with the $^8\text{B}$ being the most important. Typically ~ 3 atoms of $^{37}\text{Ar}$ are produced per week and must be isolated from the $10^{30}$ atoms of cleaning fluid in the tank, a radiochemical tour de force. The detector was placed deep underground to shield against cosmic ray background reactions.
The Davis or Cl detector was the detector used to define the “solar neutrino problem” and another type of radiochemical detectors, the SAGE/GALLEX detectors, was used to further define the problem. These detectors, GALLEX in Italy and SAGE in Russia, are based on the reaction

\[ \nu_e + ^{71}\text{Ga} \rightarrow ^{71}\text{Ge} + e^- \]

These detectors have a threshold of 0.232 MeV and can be used to directly detect the dominant pp neutrinos from the sun. The gallium is present as a solution of GaCl₃. The \(^{71}\text{Ge}\) is collected by sweeping the detector solution with \(N_2\) and converting the Ge to GeH₄ before counting. These detectors utilize 30 – 100 tons of gallium and contain a significant fraction of the world’s yearly gallium production.

The Cerenkov detectors involve the scattering of neutrinos by charged particles, causing the charged particles to emit Cerenkov radiation that can be detected by scintillation detectors. The first of these detectors were placed in a mine at Kamioka, Japan. The largest of the detectors at Kamioka is called Super Kamiokande and consists of 50,000 tons of high purity water. The detection reaction in this case is a scattering reaction

\[ \nu + e^- \rightarrow \nu + e^- \]

and the detection threshold is about 8 MeV, allowing one to observe the \(^{8}\text{B}\) neutrinos.

The Sudbury Neutrino Observatory (SNO) detector, located at Sudbury, Ontario, Canada, consists of 1000 tons of heavy water (D₂O) mounted \(\sim 2\) km below the surface in the Sudbury Nickel Mine. In addition to neutrino-electron scattering, this detector can also utilize nuclear reactions involving deuterium.
\[ \nu_e + d \rightarrow 2p + e^- \]
\[ \nu + d \rightarrow n + p + \nu \]

where, in the latter reaction, the reaction occurs for all types of neutrinos, \( \nu_e \), \( \nu_\mu \), and \( \nu_\tau \). The former reaction is sensitive to electron neutrinos only. These different types of reactions can be exploited to look for neutrino oscillations (see below). In the latter reaction, the emitted neutron is detected by an \((n,\gamma)\) reaction in which the \(\gamma\)-ray is detected by scintillation detectors. (The heavy water of the detector is surrounded by 7000 tons of ordinary water to shield against neutrons from radioactivity in the rock walls of the mine). This detector also poses radiochemical challenges as the water purity must be such that there are < 10 U or Th atoms per \(10^{15}\) water molecules.

### 12.7.4 The Solar Neutrino Problem

The solar neutrino “problem” was defined by the first results of Davis et al. using the Cl detector at the Homestake Mine. Davis et al. observed only \(\sim 1/3\) of the expected solar neutrino flux as predicted by standard models of the sun, which assume 98.5% of the energy is from the pp chain and 1.5% of the energy is from the CNO cycle. (The final result of the Cl detector experiment is that the observed solar neutrino flux is \(2.1 \pm 0.3\) SNU compared to the predicted \(7.9 \pm 2.4\) SNU, where the Solar Neutrino Unit (SNU) is defined as \(10^{-36}\) neutrino captures/s/target atom. The GALLEX and SAGE detectors subsequently reported solar neutrino fluxes of \(77 \pm 10\) SNU and \(69 \pm 13\) SNU which are to be compared to the standard solar model prediction of \(127\) SNU for the neutrinos detected by these reactions.
Such large discrepancies indicated that clearly either the models of the sun were wrong or something fundamental was wrong in our ideas of the nuclear physics involved.

12.7.5 Solution of the Problem—Neutrino Oscillations

The solution to the solar neutrino problem is that something is wrong with our ideas of the fundamental structure of matter, the so-called Standard Model. This difficulty takes the form of “neutrino oscillations” as the solution to the solar neutrino problem. (The Standard Model predicts that the three types of neutrinos are massless and that once created, they retain their identity for all time.) The basic idea is that as neutrinos come from the sun, they “oscillate”, i.e., they change from being electron neutrinos to muon neutrinos and back, etc. This oscillation is possible if there is a mass difference between the electron and muon neutrinos and that they have mass. These neutrino oscillations are enhanced by neutrino-electron interactions in the sun. It is believed that the mass $m_{\nu_e} < m_{\nu_\mu} < m_{\nu_\tau}$. The current upper limits on these masses are:

$$m_{\nu_e} < 2.2 \text{ eV}$$
$$m_{\nu_\mu} < 170 \text{ keV}$$
$$m_{\nu_\tau} < 15.5 \text{ MeV}$$

The direct observational evidence for the occurrence of neutrino oscillations came from observations with the Cerenkov detectors. The SNO detector found 1/3 the expected number of electron neutrinos coming from the sun in agreement with previous work with the radiochemical detectors. The Super Kamiokande detector, which is primarily sensitive to electron neutrinos, but has some sensitivity to other
neutrino types found \( \sim \frac{1}{2} \) the neutrino flux predicted by the standard solar models. If all neutrino types behaved similarly, the SNO and Super Kamiokande detectors should have detected the same fraction of neutrinos. Further experiments with the SNO detector operating in a mode to simultaneously detect all types of neutrinos found neutrino fluxes in agreement with the solar models. This situation is summarized in Figure 12.19

**Figure 12.19** Current status of the comparison between standard solar model predictions and experimental measurements. From Bahcall.
12.8 Synthesis of Li, Be, and B

Big Bang nucleosynthesis is responsible for the synthesis of hydrogen and helium and some of the $^7$Li. (Stellar nucleosynthesis in main sequence stars transforms about 7% of the hydrogen into $^4$He). However neither stellar nucleosynthesis or Big Bang nucleosynthesis can produce the observed abundances of Li, Be and B. Consequently the abundances of Li, Be, and B are suppressed by a factor of $10^6$ relative to the abundances of the neighboring elements. (Figure 12.2)

The extremely low abundance is the result of two factors, the relative fragility of the isotopes of Li, Be and B and the high binding energy of $^4$He, which makes the isotopes of Li, Be, and B unstable with respect to decay/reactions that lead to $^4$He. For example, the nuclei $^6$Li, $^7$Li, $^9$Be, $^{11}$B and $^{10}$B are destroyed by stellar proton irradiations at temperatures of 2.0, 2.5, 3.5, 5.0, and 5.3 x $10^6$ K, respectively. Thus these nuclei can’t survive the stellar environment. (Only the rapid cooling following the Big Bang allows the survival of the products of primordial nucleosynthesis.)

Li, Be, and B are believed to be produced in spallation reactions in which the interstellar $^{12}$C and $^{16}$O interact with protons in the galactic cosmic rays (GCR). These reactions are high-energy reactions with thresholds of 10-20 MeV. The energy spectrum of the GCR is shown in Figure 12.20
Figure 12.20  Energy spectrum of the GCR. From Audouze.

Typical cross sections for these spallation reactions are $\sim 1 - 100$ mb for $E_p > 0.1$ GeV. The time scale of the irradiation is $\sim 10^{10}$ years. The product nuclei are not subject to high temperatures after synthesis and can survive. A further testimonial to this mechanism is the relative abundances of the elements in the GCR relative to the solar abundances (Figure 12-21), which shows the enhanced yields of Li, Be, and B in the GCR. This pattern is similar to the yield distributions of the fragments from the reactions of high energy projectiles.
Figure 12-21 The relative elemental abundances in the solar system and cosmic rays. From Rolfs and Rodney.

References

Many elementary textbooks have discussions of nuclear astrophysics. We recommend the following textbooks as being especially good in their coverage.


There are a number of excellent introductory or expository treatments of nuclear astrophysics. Among those we recommend are:


Specific references cited in this chapter are


21. A comprehensive account of the solar neutrino problem and its solution is found at the website of J.N. Bahcall (http://www.sns.ias.edu/~jnb).


Problems

1. Assume an absorption cross section of $10^{-44}$ cm$^2$ for solar neutrinos interacting with matter. Calculate the probability of a neutrino interacting as it passes through the earth.

2. What is the most probable kinetic energy of a proton in the interior of the sun ($T = 1.5 \times 10^7$ K). What fraction of these protons has an energy greater than 0.5 MeV?

3. If we want to study the reaction of $^4$He with $^{16}$O under stellar conditions, what laboratory energy would we use for the $^4$He?

4. If the earth was a neutron star, what would be its radius and density?

5. If the interior temperature of the sun is $1.5 \times 10^7$ K, what is the peak energy of the $p + ^{14}$N $\rightarrow ^{15}$O + $\gamma$ reaction.

6. Which nucleosynthetic processes are responsible for the following nuclei: $^7$Li, $^{12}$C, $^{20}$Ne, $^{56}$Fe, $^{84}$Sr, $^{96}$Zr, $^{114}$Sn, $^{124}$Sn, $^{209}$Bi, $^{238}$U

7. Outline how you would construct a radiochemical neutrino detector based upon $^{115}$In.
8. Estimate the Coulomb barrier height for the following pairs of nuclei: (a) p + p (b) $^{16}$O + $^{16}$O (c) $^{28}$Si + $^{28}$Si

9. Calculate the rate of fusion reactions in the sun. Be sure to correct for the energy loss due to neutrino emission.

10. Assuming the sun will continue to shine at its present rate, calculate how long the sun will shine.

11. From the data given on the Davis detector, and the assumption that the $^{37}$Ar production rate is 0.5 atoms/day, calculate the neutrino capture rate in SNU. Assume the effective cross section for the $^8$B neutrinos is $10^{-42}$cm$^2$.

12. Calculate the evolution of the n/p ratio in the primordial universe from the information given as the temporal dependence of the temperature.

13. Make an estimate of the neutron to proton ratio in the center of the sun if the only source of neutrons is thermal equilibrium of the weak interactions.

14. Using the information on the r-process and the s-process paths in Figures 12-14 and 12-15, make estimates of the average atomic numbers of the nuclei in the peaks for N=Z in the mass abundance curves. Do the masses of these nuclei correspond to the peaks in Figure 12-4?