

TIME VALUE AND DISCOUNTING

Introduction

Money has a present value (PV), which is the value of your money today. For example, if you had \$100 in your pocket, the present value would be \$100. Money also has a future value (FV) considering compound interest, and an annual (or monthly or quarterly) value (AV), also considering interest. If you put the same \$100 in the bank at 10% interest the FV would be \$110 in one year.

To compare projected profits or costs it is important to compare equivalent measurements, such as present value versus present value. Therefore, one must be able to convert back and forth between PV, FV, and AV.

Future Value (FV) is PV or AV with compound interest credited for n years. One might want to know how much money would accumulate from a single deposit today or a fixed monthly deposit for several years in the case of retirement funds.

Present Value (PV) is FV or AV discounted to remove interest assumed to have accumulated for n years. Converting from FV to PV is termed “discounting”. If one projects profit from selling springing heifers 24 months from now, the present value would be the equivalent value of the calves sold today. The PV is always closer to zero (0) than the FV, hence “discounting”.

Annual Value (AV) is PV amortized or annualized to express a given amount as equal annual (or monthly) amounts for years 1 through n. This is identical to amortizing a loan. The borrowed amount is PV, and the payments, if annual, are AV.

There are situations where each measure is more appropriate for comparison than the others.

- Present value is the most useful measure because the maximum number of potential years does not have to be declared.
- Future value is least useful because the number of years to compound must be declared. For example, how many years for a herd expansion?
- Annual value is of intermediate use. It is applicable when an annual value is needed, such as compare a cost with annual milk income, or when alternatives are of different lengths of duration. For example, the cost of a parlor building is spread over 16 years, whereas the cost of parlor equipment is only for 8 years. So, what is the total cost? It would be correct to add the annual cost of the building to the annual cost of the equipment. It would not be correct to add their PVs or FVs because they cover different number of years.

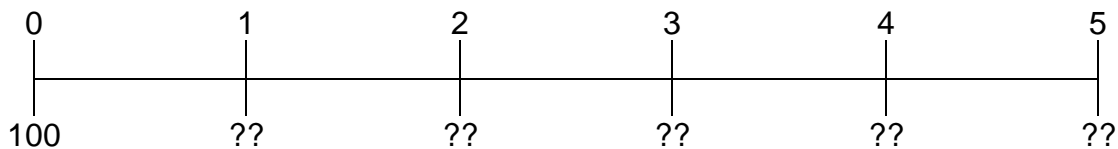
Definitions and Mechanics of Time Value Calculations

Time – The end of a year or period

MARR – Minimum Attractive Rate of Return
Nominal annual interest rate

Annual Value – Amount of money per period which is equivalent to a present or future amount.

Loan Amortization

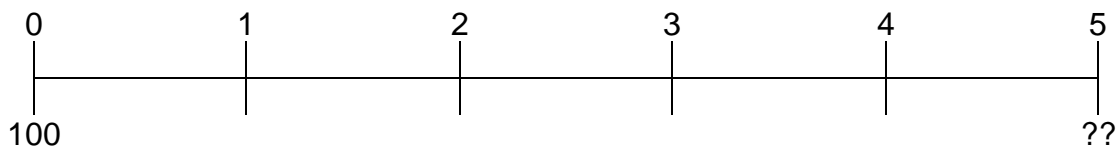


Example: 10%

$$AV = [P \rightarrow A_{(10\% 5yr)}] \times PV = [0.2637] \times 100 = 26.37$$

Future Value – Worth of money at a future point in time, including compound interest.

Compounding

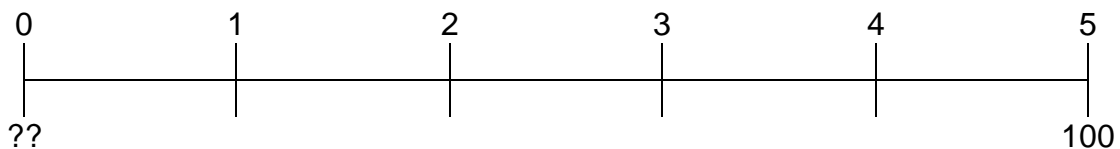


Example: 10%

$$FV = [P \rightarrow F_{(10\% 5yr)}] \times PV = [1.6105] \times 100 = 161.05$$

Present Value – Immediate worth of a future amount of money, considering compounding.

Discounting



Example: 10%

$$PV = [F \rightarrow P_{(10\% 5yr)}] \times FV = [0.6209] \times 100 = 62.09$$

The Bottom Line:

- ❑ Always draw a time line!
- ❑ Time is at end of period; start is 0
- ❑ Do not add apples and oranges - only PV to PV, AV to AV, etc.
- ❑ Do not divide or multiply dollars by 12. (That ignores monthly interest)
- ❑ First payment in annual is at time 1 (end of first period)
- ❑ PV is always closer to 0 (smaller) than FV
- ❑ If Net PV > 0 then Net AV > 0 and Net FV > 0
- ❑ At MARR=0, PV=FV and AV=PV/n=FV/n
- ❑ Net PV=0 indicates a rate of return of MARR (break even)
- ❑ Net PV >= 0 indicates acceptable profit (Rate of Return >= MARR)
- ❑ If Net PV < 0 and Raw Sum > 0, then (0% < Rate of Return < MARR)
- ❑ Inflation will reduce MARR (by subtracting from it)
- ❑ When confused, punt:
Convert all numbers to PV and then decide what you want

RULE OF 72

The Rule of 72 is that if $(\%i) \times (N) = 72$, then the future value will be double. Therefore, $(\%i) = 72/N$ or $N = 72/(\%i)$ where N equals the number of years to double.

The following will all double in value:

- 15 years at 5% interest
- 10 years at 7% interest
- 7 years at 10% interest
- 6 years at 12% interest

PV	%i	N	Years or % to Double	FV
1,000	6	?	$72/6=12$	2,000
5,000	?	8	$72/8=9\%$	10,000

You need to determine either how many years to double or find the number of years it will take to double, and translate that into how many total years (to quadruple, for instance).

Examples

If it takes someone 30 years to quadruple his/her money, then it has been doubled each $30/2=15$ years. So, $72/15=4.8\%$ interest. (4.7% actual)

If it takes someone 30 years to triple his/her money, then it got doubled more than once but not twice. So it is about $30/1.5=20$ years, giving $72/20=3.6\%$ interest. (3.7% actual)

At 12% interest, how long will it take to double your money? About 6 years ($72/12$). Therefore to quadruple will take 12 years, and octuple (8x) will take 24 years.

For the more complicated applications of the Rule of 72 you need to figure out how many years it took to double the price or amount. The number of years for a doubling is N from the formula (Total Years) = 2^N .

For instance, if something increased from 100 to 800 in 28 years, the value doubled 3 times ($8=2^3$). Therefore, the number of years it took to double the amount was $28/3 = 9$ approximately. So, $72/9 = 8\%$ /year. You can do that in your head and it is pretty close to the exact answer of 7.7%/year.