

Endogenous Timing and Strategic Choice: The Cournot-Bertrand Model

Victor J. Tremblay, Carol Horton Tremblay
and Kosin Isariyawongse*

February 16, 2011

*The corresponding author is Carol Horton Tremblay, phone: +1-541-737-1468, FAX: +1-541-737-5917, and e-mail: ctremblay@oregonstate.edu. Carol Horton Tremblay and Victor J. Tremblay are affiliated with the Department of Economics, Oregon State University, Corvallis, Oregon 97331. Kosin Isariyawongse is associated with the Department of Business and Economics, Edinboro University, Edinboro, PA 16444. The authors would like to thank Klaus Zauner and two anonymous referees for helpful comments on an earlier version of the paper.

Endogenous Timing and Strategic Choice: The Cournot-Bertrand Model

ABSTRACT. Cournot establishes a Nash equilibrium to a duopoly game under output competition; Bertrand finds a different Nash equilibrium under price competition. Both treat the strategic choice variable (output versus price) and the timing of play as exogenous. We investigate Cournot-Bertrand models where one firm competes in output and the other competes in price in both static and dynamic settings. We also develop a general model where both the timing of play and the strategic choice variable are endogenous. Consistent with the conduct of Honda and Scion, we show that Cournot-Bertrand behavior can be a Nash equilibrium outcome.

Keywords: Cournot model, Bertrand model, Stackelberg model, mixed output-price competition

JEL classifications: C72, D01, D43, L13

Endogenous Timing and Strategic Choice: The Cournot-Bertrand Model

1 Introduction

The formal study of oligopoly theory begins with the classic model of Cournot (1838). He derived the Nash equilibrium to a static duopoly game where firms produce perfectly homogeneous goods and simultaneously choose output knowing market demand and cost conditions. In equilibrium, price is below the monopoly price but above marginal cost. Bertrand (1883) analyzed the same game as Cournot except that the choice variable is price instead of output. Interestingly, Bertrand found a different Nash equilibrium, one where price equals marginal cost and firms earn zero profit. Thus, these simple models demonstrate that the choice of output or price as the strategic variable can have a dramatic effect on the Nash outcome in an oligopoly market where there is strategic interaction.

Fisher (1898) was concerned with the static nature of the Cournot and Bertrand models, arguing that real firms engage in various forms of dynamic behavior. Later, Stackelberg (1934) identified the subgame perfect Nash equilibrium (SPNE) to a dynamic Cournot model, where one firm produces output in the first period and the other firm produces output in the second period. A natural extension of this model is the dynamic Bertrand game, where one firm chooses price in the first period and the

other chooses price in the second period. These models show how dynamic considerations can give one firm a strategic advantage over the other. In Stackelberg, the first mover earns higher profits, while in dynamic Bertrand the second mover earns higher profits.

More recent work extends these models in two ways. The first allows the choice of strategic variable to be endogenous, which can result in a Cournot or a Bertrand outcome. It also allows for the possibility of a Cournot-Bertrand model, where one firm competes in output and the other firm competes in price. The second extension allows the timing of play to be endogenous. In the case of a two period model, firms have the option of acting in the early period or the late period.

Table 1 outlines the relevant models and their important characteristics: (1) the choice of strategic variable, whether firms compete in output, price, or have endogenous choices, and (2) the timing of play, which can be static, dynamic, or endogenous. The standard games are labeled Cournot, Bertrand, Dynamic Cournot (Stackelberg), and Dynamic Bertrand. The static Cournot-Bertrand model is identified as M1 and the dynamic Cournot-Bertrand model is identified as M3. The remaining models and our contribution to the literature will be discussed shortly.

<Table 1 about here>

Singh and Vives (1984) formally explored the possibility of a static Cournot-Bertrand model when the choice of strategic variable is

endogenous, labeled model M2 in Table 1. They find that the dominant strategy is for both firms to compete in output. That is, the Cournot outcome dominates the Bertrand and Cournot-Bertrand outcomes. As a result, they do not investigate the characteristics of the Cournot-Bertrand model (model M1).¹

Nevertheless, Cournot-Bertrand behavior occurs in the real world. In the U.S. market for small cars, for example, Scion (and Saturn) dealers behave as Bertrand-type firms, while Honda (and Subaru) dealers behave as Cournot-type firms.² That is, Honda dealers set quantities (i.e., monthly inventories) and adjust prices to clear the market, while Scion dealers set prices and then supply cars to customers only after orders are placed. This example suggests that further study is needed regarding the Cournot-Bertrand model.³

Hamilton and Slutsky (1990), Amir (1995), and Amir and Grilo (1999) have developed models that allow the timing of play to be endogenous, although the strategic choice variable remains exogenous. They

¹With vertical product differentiation, Häckner (2000) and Hsu and Wang (2005) show that price competition can be more profitable than output competition when the quality difference between brands is sufficiently large. d'Aspremont and Ferreira (2009) investigate a general model that nests a wide variety of oligopoly theories but do not consider the mixed Cournot-Bertrand case. Tremblay and Tremblay (forthcoming) investigate the effect of product differentiation on the Nash equilibrium in a Cournot-Bertrand model when each firm's strategic variable is exogenously determined.

²Another example is the aerospace connector industry in the U.S. where leading distributors act as Bertrand-type firms and smaller distributors act as Cournot-type firms.

³This follows from Kreps and Scheinkman's (1983) argument that it is "witless" to choose one static oligopoly model (e.g., Cournot) over another (Bertrand), as it is an empirical question whether or not firms compete in output or price.

focused on models M5 and M6 (Table 1). Hamilton and Slutsky also briefly discuss model M7. In a model with two firms and two periods (early and late), two types of outcomes are possible: (1) Static, where both firms act in the early period or both firms act in the later period, and (2) Dynamic, where one firm acts early and the other firm acts late. This new body of research shows that the SPNE in an output game is for both firms to act simultaneously in the early period. In a pricing game, there are multiple equilibria with each player preferring a dynamic setting.⁴

In spite of these extensions, the literature is deficient in a number of ways. First, the possibility of a Cournot-Bertrand model has not been adequately investigated (model M1). In addition, the dynamic Cournot-Bertrand model (M3) and the dynamic model where the choice of strategic variable is endogenous (M4) have not yet been discussed. Finally, the case where both the strategic variable and the timing of play (early or late) are endogenous has not been analyzed (M8).

We attempt to fill these gaps in the literature. First, we revisit models M1 and M2 to show that the static Cournot-Bertrand outcome is optimal when there are asymmetric costs. Given this finding and the fact that such behavior is observed in the real world, we also describe the key features of

⁴These models assume that both firms are privately owned. When one firm is private and the other firm is public in an output game, however, Matsumura (2003) shows that the SPNE is Stackelberg, with the public firm choosing output in the early period and the private firm choosing output in the late period. Romano and Yildirim (2005) extend this line of research by allowing two private firms to continuously adjust production or price targets. Neither study considers mixed Cournot-Bertrand behavior.

the Cournot-Bertrand model that have not previously been discussed. Second, we investigate the possibility of dynamic Cournot-Bertrand models (M3 and M4). We find that when the choice of strategic variable is endogenous in a dynamic setting, the SPNE is just as likely to be a dynamic Cournot-Bertrand outcome as it is a Stackelberg or dynamic Bertrand outcome. Finally, in the general case where both the timing and strategic variable are endogenous (M8), we show that the dynamic Cournot-Bertrand outcome can be an SPNE.

2 The Static Case with Endogenous Strategic Choice

Because there are examples of Cournot-Bertrand behavior in the real world, one of our goals is to identify conditions that support it and describe its characteristics. The models in this and subsequent sections assume that information is perfect and complete.

2.1 Justification for the Cournot-Bertrand Model

Following Singh and Vives, we assume that two profit maximizing firms (1 and 2) produce differentiated products and face negatively sloped demand functions.⁵ Marginal cost is normalized to zero. The endogenous choice of

⁵The best-reply functions normally have a negative slope when both firms compete in output (Cournot) and have a positive slope when both compete in price. Although

the strategic variable is modeled by assuming that firms have the option of choosing to compete in output or price in a preplay stage of the game. Once this decision is made, they then simultaneously choose the optimal level of their strategic variable. This leads to four possible outcomes:

1. Cournot (CC): both firms compete in output.
2. Bertrand (BB): both firms compete in price.
3. Cournot-Bertrand (CB): firm 1 competes in output and firm 2 competes in price.
4. Bertrand-Cournot (BC): firm 1 competes in price and firm 2 competes in output.

Nash equilibrium profits for each outcome are defined as π_i^k , where subscript i refers to firm 1 or 2 and superscript k refers to the particular outcome: CC, BB, CB, or BC. In a game such as this, Singh and Vives prove that the dominant strategy for each firm is Cournot (their Proposition 2). That is, $\pi_1^{CC} > \pi_1^{BC}$ and $\pi_1^{CB} > \pi_1^{BB}$ for firm 1, and $\pi_2^{CC} > \pi_2^{CB}$ and $\pi_2^{BC} > \pi_2^{BB}$ for firm 2. However, one way to support the Cournot-Bertrand model is to assume that the choice of strategic variable is exogenously given, as is done in the traditional Cournot and Bertrand

there are cases where these sign conditions need not hold, Amir and Grilo (1999) call this the “typical geometry” for the Cournot and Bertrand models. We assume this typical geometry for the remainder of the paper.

models. For example, institutional constraints might require that firm 1 compete in output and firm 2 compete in price.

Asymmetric set-up or fixed costs can also induce a Cournot-Bertrand outcome. Assume that there are fixed costs only when a firm competes in output and that firm 2's fixed costs (F_2) exceed firm 1's fixed costs (F_1). For convenience, we normalize F_1 to zero. Under these conditions, the Cournot-Bertrand outcome becomes the Nash equilibrium when F_2 is sufficiently high. That is, firm 1's dominant strategy remains to compete in output, but firm 2 prefers to compete in price because $\pi_2^{CB} > \pi_2^{CC} - F_2$ and $\pi_2^{BB} > \pi_2^{BC} - F_2$, where π_2^{CC} and π_2^{BC} are variable profits. Thus, fixed costs can affect a firm's choice of strategic variable.

According to Palmeri et al. (2003), asymmetric fixed costs explain the strategic choices in the market for small cars in the U.S. Fixed costs are higher for output competition than price competition, because output competition requires a dealer to hold a large inventory relative to a firm that competes in price and ships only after an order is placed. Scion (firm 2) is a relatively new brand that is distributed by Toyota dealers. When Scion was introduced into the U.S. in 2003, existing Toyota dealers faced a capacity constraint, with sales lots full of new Toyotas and used cars. Choosing to compete in price and shipping to order kept Scion's fixed costs relatively low. This is why Scion prefers price competition. In contrast, Honda (firm 1) has established dealers that have competed in output for decades. Switching to price competition would incur a switching cost, as

Honda has existing storage capacity which would be costly to liquidate. With sufficiently high switching costs or sufficiently low fixed costs associated with holding inventory, Honda sticks with output competition.

2.2 General Properties of the Cournot-Bertrand Model

Given that the static Cournot-Bertrand outcome can be optimal, we now describe the characteristics of the model. We consider a differentiated duopoly game where firm 1 competes in output, $q_1 \in [0, \infty)$ and firm 2 competes in price, $p_2 \in [0, \infty)$. Firm i 's profit function depends on both strategic variables, $\pi_i(q_1, p_2)$, and is concave and twice continuously differentiable. Best-reply functions are strictly monotone, and an equilibrium exists on the interior of the action space.

Even though demand and cost functions may be symmetric, the model has a natural asymmetry. Firm 1's profit function exhibits increasing marginal returns to p_2 (that is, q_1 and p_2 are strategic complements) and firm 2's profit function exhibits decreasing marginal returns to q_1 (that is, q_1 and p_2 are strategic substitutes). In other words, $\pi_{12} > 0$ and $\pi_{21} < 0$, where π_{ij} is defined as the second derivative of firm i 's profit function with respect to firm i 's choice variable and its rival's choice variable.

If cooperation were possible, this asymmetry influences each firm's incentive to cheat on the cartel agreement in different ways. With an

enforceable contract to maximize joint profits, $\Pi = \pi_1 + \pi_2$, the first-order conditions are

$$\frac{\partial \Pi}{\partial q_1} = \frac{\partial \pi_1}{\partial q_1} + \frac{\partial \pi_2}{\partial q_1} = 0 \quad (1)$$

$$\frac{\partial \Pi}{\partial p_2} = \frac{\partial \pi_1}{\partial p_2} + \frac{\partial \pi_2}{\partial p_2} = 0. \quad (2)$$

Because $\partial \pi_2 / \partial q_1 < 0$, $\partial \pi_1 / \partial q_1$ must be positive, and firm 1 has an incentive to cheat on the cartel agreement by increasing q_1 . Similarly, $\partial \pi_2 / \partial p_2$ must be negative because $\partial \pi_1 / \partial p_2 > 0$. Thus, firm 2's incentive is to cheat by lowering p_2 .

In a non-cooperative setting, the structure of the model guarantees a unique Nash equilibrium. If we define firm 1's best-reply function as $r_1(p_2) = \operatorname{argmax} \pi_1(q_1, p_2)$ and firm 2's best-reply function as $r_2(q_1) = \operatorname{argmax} \pi_2(p_2, q_1)$, then the Nash equilibrium occurs where the best-reply functions intersect. From the first-order conditions and the implicit-function theorem, the slope of r_1 is positive and the slope of r_2 is negative.⁶ This along with continuity of the best-reply functions guarantees uniqueness.

We illustrate these results with linear demand and cost functions that are similar to those found in Singh and Vives (1984). Variable costs are

⁶The slope of r_1 equals $-\pi_{12}/\pi_{11}$ which is positive because $\pi_{12} > 0$ and $\pi_{11} < 0$ (from concavity). The slope of r_2 equals $-\pi_{21}/\pi_{22}$, which is negative because $\pi_{21} < 0$ and $\pi_{22} < 0$ (from concavity). Stability requires that when out of equilibrium, the dynamic (myopic) adjustment process converges to the Nash equilibrium. According to Dixit (1986), stability requires that $|\partial^2 \pi_1 / \partial q_1^2| > |\partial^2 \pi_1 / \partial q_1 \partial p_2|$ and $|\partial^2 \pi_2 / \partial p_2^2| > |\partial^2 \pi_2 / \partial p_2 \partial q_1|$.

normalized to zero. With no fixed costs, the profit functions are $\pi_1(q_1, p_2) = p_1 q_1$ and $\pi_2(q_1, p_2) = p_2 q_2$. This is accurate when firm 2 competes in price where $F_2 = 0$, but if firm 2 were to compete in output, its full profit would be $\pi_2 - F_2$, where $F_2 > 0$. Demand functions in choice variables are⁷

$$p_1(q_1, p_2) = a - q_1 + b p_2 \quad (3)$$

$$q_2(q_1, p_2) = a - p_2 - d q_1, \quad (4)$$

where $a \in (0, \infty)$, $b \in (0, \frac{2}{3})$, and $d \in (0, 2)$.⁸

The Nash equilibrium values in the static Cournot-Bertrand model are

$$p_1^* = \frac{a(2+b)}{4+bd} > p_2^* = \frac{a(2-d)}{4+bd} \quad (5)$$

$$q_1^* = \frac{a(2+b)}{4+bd} > q_2^* = \frac{a(2-d)}{4+bd} \quad (6)$$

$$\pi_1^* = \frac{a^2(2+b)^2}{(4+bd)^2} > \pi_2^* = \frac{a^2(2-d)^2}{(4+bd)^2}. \quad (7)$$

The model has a natural asymmetry, with the strategic advantage going to

⁷Assume that $a_1 = (a + ab)$, $a_2 = (a - ad)$, $a_3 = a_1/d_1$, $a_4 = a_2/d_1$, $b_3 = b/d_1$, $d_1 = (1 + bd)$, $d_3 = 1/d_1$, and $d_4 = d/d_1$. This structure produces the following inverse demand functions in the CC model: $p_1 = a_1 - d_1 q_1 - b q_2$ and $p_2 = a - q_2 - d q_1$. In the BB model, the demand functions are $q_1 = a - p_1 + b p_2$ and $q_2 = a_2 - b_2 p_2 + d p_1$. In the BC model, the demand functions are $q_1 = a_3 - d_3 p_1 - b_3 q_2$ and $p_2 = a_4 - d_3 q_2 + d_4 p_1$.

⁸These parameter restrictions ensure firm participation for all possible strategic choice combinations when the model is static and dynamic.

firm 1.⁹

The equilibrium is illustrated in Figure 1, which graphs the best-reply and isoprofit functions assuming that $a = 19$, $b = 0.5$, and $d = 1.5$. Because firm 1's best reply has a positive slope and firm 2's has a negative slope, the Nash equilibrium is unique at $p_1^* = q_1^* = 10$, $p_2^* = q_2^* = 2$, $\pi_1^* = 100$, and $\pi_2^* = 4$.

<Figure 1 about here>

Nash equilibria for the other three cases (CC, BB, and BC) are determined similarly. When there are zero fixed costs and the choice of strategic variable is endogenous, each firm's dominant strategy is to compete in output. When given the option, firms will always compete in output in a static setting. Table 2 illustrates this for the example above. But when firm 2 competes in output and $10.4 \leq F_2 < 11.2$, the Cournot-Bertrand outcome is the Nash equilibrium. Firm 1's dominant strategy remains the same, but firm 2's dominant strategy is now to compete in price. This illustrates that the Cournot-Bertrand outcome is possible when firms have asymmetric fixed costs.

<Table 2 about here>

⁹Recall that F_1 was normalized to zero. If F_1 were positive and sufficiently high, firm 2 would have the strategic advantage. When variable costs are greater than zero, demand is log-convex, and the model is dynamic Cournot, Amir and Grilo (1999) find that firm 1 (the leader) has a strategic disadvantage.

3 The Dynamic Case with Endogenous

Strategic Choice

In this section, we investigate the dynamic leader-follower models when the choice of strategic variable is endogenous. Here, one firm (the leader) makes an output or price decision in an early period (E) and the other firm (the follower) makes an output or price decision in a late period (L). This setup can produce dynamic Cournot, dynamic Bertrand, and dynamic Cournot-Bertrand (M3) models, as listed in Table 1. The possibilities are:

1. Firm 1 chooses output early and firm 2 chooses output late (q_{1E}, q_{2L});
firm 1 chooses output early and firm 2 chooses price late (q_{1E}, p_{2L}).
2. Firm 1 chooses price early and firm 2 chooses output late (p_{1E}, q_{2L});
firm 1 chooses price early and firm 2 chooses price late (p_{1E}, p_{2L}).
3. Firm 2 chooses output early and firm 1 chooses output late (q_{2E}, q_{1L});
firm 2 chooses output early and firm 1 chooses price late (q_{2E}, p_{1L}).
4. Firm 2 chooses price early and firm 1 chooses output late (p_{2E}, q_{1L});
firm 2 chooses price early and firm 1 chooses price late (p_{2E}, p_{1L}).

An interesting outcome in this case is that once the leader has made its choice of competing in output or price, the payoff to the follower is the same whether the follower chooses to compete in output or in price. Thus,

of the 8 possibilities, only 4 are distinct. We state this result formally in Proposition 1.

Proposition 1 *Firm i chooses its action, q_i or p_i , in the first period, which is observed by firm j . In the second period, firm j maximizes its profit with respect to its action, q_j or p_j . Each firm's demand is continuous and has a negative slope. Demand is positive when the price is below some positive but finite number x , and demand is zero for $p_i \geq x$. Each firm's cost function is twice continuously differentiable and strictly convex. Fixed costs are zero. A unique equilibrium exists on the interior of the action space. Under these conditions, firm j 's profits will be the same whether firm j competes in output or price.*

Proof. Consider the case where firm i moves first and competes in output. In the second period, firm j chooses its optimal level of output or price. If it competes in output, its problem is to

$$\max_{q_j} \pi_j = p_j(q_i, q_j)q_j - C(q_j), \quad (8)$$

where $p_j(q_i, q_j)$ is firm j 's inverse demand function and $C(q_j)$ is its cost function. Because q_i is predetermined and equal to a constant in the second period, firm j 's (residual) demand function is only a function of q_j . The firm's first-order condition is

$$\frac{\partial \pi_j}{\partial q_j} = p_j + \frac{\partial p_j}{\partial q_j} q_j - \frac{\partial C}{\partial q_j} = 0. \quad (9)$$

If instead firm j competes in price, its problem is to

$$\max_{p_j} \pi_j = p_j q_j(q_i, p_j) - C[q_j(q_i, p_j)], \quad (10)$$

where $q_j(q_i, p_j)$ is firm j 's demand function. Again, because q_i is predetermined and equal to a constant, firm j 's (residual) demand function is only a function of p_j . The firm's first-order condition is

$$\frac{\partial \pi_j}{\partial p_j} = q_j + \frac{\partial q_j}{\partial p_j} p_j - \frac{\partial C}{\partial q_j} \frac{\partial q_j}{\partial p_j} = 0. \quad (11)$$

Multiplying both sides of equation (11) by $\partial p_j / \partial q_j$ produces equation (9). Thus, the first-order conditions and the optimal values of p_j and q_j will be the same whether firm j competes in price or quantity. A similar argument applies when firm i competes in price in period 1. ■

The intuition behind Proposition 1 is that firm i sets q_i (or p_i) equal to a constant in the first period. In the second period, firm j faces a residual demand function that depends only on its own action. Therefore, the structure of firm 2's problem is just like that of a monopolist, where the monopoly outcome is the same whether the firm maximizes profit with respect to output or price. Proposition 1 demonstrates that when the strategic variable is endogenous, the dynamic Cournot-Bertrand outcome and the dynamic Bertrand-Cournot outcome are SPNE; they are just as likely to occur as the traditional Stackelberg and dynamic Bertrand

outcomes. Note that asymmetric fixed costs are not needed to support a Cournot-Bertrand or a Bertrand-Cournot outcome in a dynamic setting.

4 A Model with Endogenous Timing and Strategic Choice

In this section, we investigate a general duopoly model where both the timing and choice of strategic variable are endogenous. Each firm has the choice of competing in output or price and has the option of acting in an early or late stage of the game. Other characteristics of the game remain the same as in the previous section.

There are 16 possible outcomes in the extensive form of the game. In the static cases where both firms act in the same period and choose the same strategic variable, there is one early and one late Cournot outcome (E-CC and L-CC) and one early and one late Bertrand outcome (E-BB and L-BB). Similarly, if different strategic variables are chosen, this leads to the mixed Cournot-Bertrand cases: E-CB, L-CB, E-BC, and L-BC. Assuming no discounting, only 4 of the 8 static cases are distinct:

1. Both firms choose output, early and late (E-CC and L-CC).
2. Both firms choose price, early and late (E-BB and L-BB).
3. Firm 1 chooses output and firm 2 chooses price, early and late (E-CB and L-CB).

4. Firm 1 chooses price and firm 2 chooses output, early and late (E-BC and L-BC).

The 8 dynamic cases are described in the previous section.

Before discussing the fully endogenous case, it is useful to note an interesting feature of a dynamic game in a Cournot-Bertrand setting (model M7 in Table 1). In this case, both firms are better off if the Cournot-type firm moves early and the Bertrand-type firm acts late.¹⁰ This is evident from Figure 1. If firm 1 moves first, it will maximize its profit given the best reply of firm 2, which will occur in the Pareto superior region (i.e., the shaded region) of the diagram. In our numerical example above, by moving dynamically rather than statically firm 1's profits increase from 100 to 102.6, and firm 2's profits increase from 4 to 9.1. Thus, when the timing of play is endogenous in a Cournot-Bertrand setting, firms will prefer to play a dynamic rather than a static game.

Finally, we consider the case where both the timing and the choice of strategic variable are endogenous (model M8). The results for this case given different assumptions about fixed costs are presented in Propositions 2 and 3.

Proposition 2 *Firms compete in the general linear game described in Section 2.2 when fixed costs are zero. Firms have the choice of strategic*

¹⁰Although they do not explicitly discuss the Cournot-Bertrand model, Hamilton and Slutsky (1990) and Amir (1995) observe that when firm i 's best reply has a positive slope and firm j 's best reply has a negative slope, both firms are better off if firm i chooses to act early and firm j chooses to act late.

variable (output or price) and the timing of play (early or late). Under these conditions, the SPNE is the early Cournot outcome, where both firms compete in output in the early period.¹¹

Proof. See the Appendix.

This conclusion can be overturned, however, with the addition of appropriately defined fixed costs.

Proposition 3 *Firms compete in the same game as described in Proposition 2, except that firm 2 faces a positive fixed cost when competing in output (F_2). When F_2 is sufficiently large, the SPNE is the dynamic Cournot-Bertrand outcome, where firm 1 competes in output early and firm 2 competes in price late.*

Proof. See the Appendix

In this setting, firm 1's optimal choice remains q_{1E} , while firm 2's optimal choice switches from q_{2E} to p_{2L} . Thus, when both the strategic variable and the timing of play are endogenous, a dynamic Cournot-Bertrand outcome can be an SPNE. This provides another possible explanation for the behavior of Honda and Scion, as Honda is the established firm that competes in output and Scion is the later entrant that competes in price.

¹¹This is an iterated-dominant outcome, as only firm 1 has a dominant strategy for all permissible parameter values: $a > 0$, $0 < b < 2/3$, and $0 < d < 2$.

5 Concluding Remarks

Unlike previous research, we develop a duopoly model that allows both the timing of play and the choice of strategic variable to be endogenous decisions. Our model provides a justification for the observation that within an industry some firms compete in output and others compete in price, as in the U.S. market for small cars. Unfortunately, this model, which we call Cournot-Bertrand, has received little attention in the literature. As a result, we investigate the properties of the model in both static and dynamic settings and in the more general case where the strategic variable (output or price) and timing of play (early or late) are endogenous.

Three noteworthy findings emerge from our analysis. First, when firms compete in a static game and the choice of strategic variable is endogenous, the Cournot-Bertrand outcome can be a Nash equilibrium with appropriately defined set-up costs. Second, when firms compete in a dynamic game but have a choice of strategic variable, a dynamic Cournot-Bertrand outcome is an SPNE. That is, firms are just as likely to be Cournot-Bertrand competitors as Stackelberg competitors. This holds even when fixed costs are zero. Finally, when both the strategic variable and the timing of play are endogenous and set-up costs are asymmetric, the dynamic Cournot-Bertrand outcome can be an SPNE. Thus, there is considerable validation for the static and dynamic versions of the Cournot-Bertrand model.

Appendix

The proof of Proposition 2 is as follows. Note that several steps in the proof are based on our Proposition 1 above, Theorem V in Hamilton and Slutsky (1990) and Amir and Grilo (1999), and Proposition 2 in Singh and Vives (1984).

1. Firm 1's dominant strategy is to play output early (q_{1E}). The argument is as follows.

(a) When firm 2 chooses output early (q_{2E}), firm 1's choice of q_{1E} dominates:

- i. output late (q_{1L}) (Hamilton and Slutsky; Amir and Grilo).
- ii. price early (p_{1E}) (Singh and Vives).
- iii. price late (p_{1L}), because q_{1E} dominates q_{1L} , and firm 1's profit is the same whether it competes in q_{1L} or p_{1L} (Proposition 1). Therefore, q_{1E} dominates p_{1L} .

(b) When firm 2 chooses output late (q_{2L}), q_{1E} dominates:

- i. q_{1L} (Hamilton and Slutsky; Amir and Grilo 1999).
- ii. p_{1E} . For these alternatives, firm 1's profits are:

$$\pi_1(q_{1E}, q_{2L}) = [a^2(2+b)^2]/[8(2+bd)], \text{ and}$$

$$\pi_1(p_{1E}, q_{2L}) = [a^2(2+b+bd)^2]/[8(2+3bd+b^2d^2)].$$

$$\pi_1(q_{1E}, q_{2L}) - \pi_1(p_{1E}, q_{2L}) =$$

$[a^2b^2d(2 + b - d)]/[8(2 + 3bd + b^2d^2)]$, which is positive

because d must be less than 2.

- iii. p_{1L} . From above, q_{1E} dominates q_{1L} , and q_{1L} dominates p_{1L} (Singh and Vives). Therefore, q_{1E} dominates p_{1L} .

(c) When firm 2 chooses price early (p_{2E}), q_{1E} dominates:

- i. q_{1L} . For these alternatives, firm 1's profits are:

$$\pi_1(q_{1E}, p_{2E}) = [a^2(2 + b)^2]/(4 + bd)^2, \text{ and}$$

$$\pi_1(q_{1L}, p_{2E}) = [a^2(4 + b(2 + d))^2]/[16(2 + bd)^2].$$

$$\pi_1(q_{1E}, p_{2E}) - \pi_1(q_{1L}, p_{2E}) =$$

$$\{a^2b^2d(2 - d)[32 + 16b(1 + d) + b^2d(6 + d)]\}/[16(8 + 6bd + b^2d^2)^2],$$

which is positive because d must be less than 2. Therefore,

q_{1E} dominates q_{1L} .

- ii. p_{1E} (Singh and Vives).
- iii. p_{1L} , because q_{1E} dominates q_{1L} and firm 1's profit is the same whether it competes in q_{1L} or p_{1L} (Proposition 1).
Therefore, q_{1E} dominates p_{1L} .

(d) When firm 2 chooses price late (p_{2L}), q_{1E} dominates:

- i. q_{1L} (Hamilton and Slutsky; Amir and Grilo).
- ii. p_{1E} . $\pi_1(q_{1E}, q_{2L}) > \pi_1(p_{1E}, q_{2L})$ from b.ii above.
 $\pi_1(q_{1E}, q_{2L}) = \pi_1(q_{1E}, p_{2L})$ and $\pi_1(p_{1E}, q_{2L}) = \pi_1(p_{1E}, p_{2L})$
from our Proposition 1. Therefore, q_{1E} dominates p_{1E} .
- iii. p_{1L} , because q_{1E} dominates q_{1L} (Hamilton and Slutsky; Amir

and Grilo) and q_{1L} dominates p_{1L} (Singh and Vives).

2. Given that firm 1's dominant strategy is to play q_{1E} , firm 2's best reply is to play q_{2E} . That is, when firm 1 chooses q_{1E} , competing in q_{2E} dominates:

- (a) q_{2L} (Hamilton and Slutsky; Amir and Grilo).
- (b) p_{2E} (Singh and Vives).
- (c) p_{2L} , because q_{2E} dominates q_{2L} and firm 2's profit is the same whether it competes in q_{2L} or p_{2L} (Proposition 1). ■

The proof of Proposition 3 is as follows. Because fixed costs are only imposed on firm 2 (F_2), firm 1's dominant strategy remains q_{1E} (Proposition 2). Ignoring fixed costs for the moment, the following profit conditions hold for firm 2: $\pi_2(q_{1E}, q_{2E}) > \pi_2(q_{1E}, q_{2L}) = \pi_2(q_{1E}, p_{2L}) > \pi_2(q_{1E}, p_{2E})$. This is because $\pi_2(q_{1E}, q_{2E}) > \pi_2(q_{1E}, q_{2L})$ by Proposition 2, $\pi_2(q_{1E}, q_{2L}) = \pi_2(q_{1E}, p_{2L})$ by Proposition 1, and $\pi_2(q_{1E}, p_{2L}) > \pi_2(q_{1E}, p_{2E})$ by Hamilton and Slutsky (1990) and by Amir and Grilo (1999). With positive fixed costs when firm 2 competes in output, this condition becomes $\pi_2(q_{1E}, q_{2E}) - F_2 > \pi_2(q_{1E}, q_{2L}) - F_2 < \pi_2(q_{1E}, p_{2L}) > \pi_2(q_{1E}, p_{2E})$. When F_2 is sufficiently high, $\pi_2(q_{1E}, p_{2L}) > \pi_2(q_{1E}, q_{2E}) - F_2$. Thus, firm 2's best reply to firm 1's decision to compete in q_{1E} is to compete in p_{2L} . ■

References

- Amir, Rabah, "Endogenous Timing in Two-Player Games: A Counterexample," *Games and Economic Behavior*, 9 (2), May 1995, 234-237.
- Amir, Rabah, and Grilo, Isabel, "Stackelberg versus Cournot Equilibrium," *Games and Economic Behavior*, 26 (1), January 1999, 1-21.
- Amir, Rabah, and Jin, Jim Y., "Cournot and Bertrand Equilibria Compared: Substitutability, Complementarity and Concavity," *International Journal of Industrial Organization*, 19(3-4), March 2001, 303-317.
- Bertrand, Joseph, "Review," *Journal des Savants*, 68, 1883, 499-508.
- Cournot, Augustin, *Researches into the Mathematical Principles of the Theory of Wealth*, Paris: L. Hachette, 1838.
- d'Aspremont, Claude, and Ferreira, Rodolphe Dos Santos, "Price-Output Competition with Varying Toughness," *Games and Economic Behavior*, 65 (1), January 2009, 62-82.
- De Fraja, Giovanni, and Delbono, Flavio, "Game Theoretic Models of Mixed Oligopoly," *Journal of Economic Surveys*, 4 (1), 1990, 1-17.
- Dixit, Avinash, "Comparative Statics for Oligopoly," *International Economic Review*, 27 (1), February 1986, 107-122.
- Fisher, Irving, "Cournot and Mathematical Economics," *Quarterly Journal of Economics*, 12 (2), January 1898, 119-138.

Häckner, Jonas, "A Note on Price and Quantity Competition in Differentiated Oligopolies," *Journal of Economic Theory*, 93 (2), August 2000, 233-239.

Hamilton, Jonathan and Slutsky, Steven, "Endogenous Timing in Duopoly Games: Stackelberg or Cournot Equilibria," *Games and Economic Behavior*, 2 (1), March 1990, 29-46.

Hsu, Judy and Wang, X. Henry, "On Welfare under Cournot and Bertrand Competition in Differentiated Oligopolies," *Review of Industrial Organization*, 27 (2), September 2005, 185-191.

Kreps, David, and Scheinkman, Jose, "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," *Bell Journal of Economics*, 14 (2), Autumn 1983, 326-337.

Matsumura, Toshihiro, "Stackelberg Mixed Duopoly with a Foreign Competitor," *Bulletin of Economic Research*, 55 (3), June 2003, 275-287.

Palmeri, Christopher, Elgin, Ben, and Kerwin, Kathleen, "Toyota's Scion: Dude, Here's Your Car," *Business Week*, June 9, 2003.

Romano, Richard, and Yildirim, Huseyin, "On the Endogeneity of Cournot-Nash and Stackelberg Equilibria: Games of Accumulation," *Journal of Economic Theory*, 120 (1), January 2005, 73-107.

Singh, Nirvikar, and Vives, Xavier, "Price and Quantity Competition in a Differentiated Duopoly," *Rand Journal of Economics*, 15 (4), Winter 1984, 546-554.

Stackelberg, Henrich von, *Marktform und Gleichgewicht*, Vienna:

Julius Springer, 1934.

Tremblay, Carol Horton, and Tremblay, Victor J., “The Cournot-Bertrand Model and the Degree of Product Differentiation,” *Economics Letters*, forthcoming.

Vives, Xavier, *Oligopoly Pricing: Old Ideas and New Tools*, Cambridge, MA: MIT Press, 1999.

TABLE 1. Duopoly Models: Output and Price Competition and in Static and Dynamic Settings

	Output	Strategic Variable *		Endogenous (Output or Price)
		Price	Output-Price	
Timing: **				
Static	Cournot	Bertrand	<u>M1</u>	<u>M2</u>
Dynamic	Dynamic-Cournot (Stackelberg)	Dynamic-Bertrand	M3	M4
Endogenous (Early or Late)	M5	M6	<u>M7</u>	M8

Note: Models in bold have not been discussed in the literature (M3, M4, and M8). Underlined models have been briefly discussed but have been inadequately developed in the literature (M1, M2, and M7).

* Output means that both firms compete in output; Price means that both firms compete in price; Output-Price means that one firm competes in output and the other firm competes in price; Endogenous means that firms can choose whether to compete in output or price.

** Static means that the game is static; Dynamic means that the game is dynamic; Endogenous means that firms choose whether to compete in an early or late period.

TABLE 2. Nash Equilibrium Profits for Cournot (CC), Bertrand (BB), and Bertrand-Cournot (BC) Outcomes

		Firm 2	
		q_2	p_2
Firm 1	q_1	$\pi_1^{CC} = 101.1, \pi_2^{CC} = 14.4 - F_2$	$\pi_1^{CB} = 100, \pi_2^{CB} = 4$
	p_1	$\pi_1^{BC} = 88, \pi_2^{BC} = 11.2 - F_2$	$\pi_1^{BB} = 97.6, \pi_2^{BB} = 4.04$

FIGURE 1

Best Reply and Isoprofit Functions in a Cournot-Bertrand Duopoly

