

**Oregon State University  
Department of Economics**

**Microeconomic Theory II  
Homework 1**

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**Instructions:** Homework 1 is worth 50 points and each question is equally weighted. You are encouraged to work together with other students, but please write up your answers independently and submit an answer only if you understand it.

1. Assume a perfectly competitive market where each firm faces the following long-run total cost function.

$$\text{(Total Cost Function) } TC = 35q - 5q^2 + q^3/4.$$

The market demand function is:

$$\text{(Demand) } Q = 1210 - p,$$

where  $p$  is price and  $Q$  is the sum of each firm's output ( $q$ ).

- A. Determine the long-run equilibrium price and output ( $q$ ) for the representative firm.
- B. Determine the long-run equilibrium level of output in the market ( $Q$ ).
- C. Determine the long-run equilibrium number of firms ( $n$ ).

2. Assume a constant cost industry with the following demand and cost conditions.

$$\text{(Inverse Demand) } p = a - bQ$$

$$\text{(Total Cost) } TC = c q$$

where  $p$  is price,  $Q$  is market output,  $q$  is the firm's output. Parameters  $a$ ,  $b$ , and  $c$  are positive, and  $a > c$ .

- A. Determine the equilibrium price and market output if this market is perfectly competitive.
- B. Determine the equilibrium price and market output if there is just one firm in the industry (i.e.,  $q = Q$ ).
- C. If social welfare is defined as producer plus consumer surplus (i.e., there are no income effects), define and use this function to determine the socially optimal price and output.
- D. Determine the deadweight loss due to monopoly and show how it is affected by changes in the parameters of the model (i.e.,  $a$ ,  $b$ , and  $c$ ).
- E. Compare the impact of an excise tax ( $t$  per unit) on the proportion of the tax paid by consumers when the market is perfectly competitive versus a monopoly. How is the slope of demand affect your answer?

3. Assume a monopolist faces the following demand and cost functions.

$$\text{(Demand) } q_D = 100/p$$

$$\text{(Total Cost) } TC = q^2.$$

What is the optimal level of output for the monopolist?

4. Assume a monopolist is able to effectively price discriminate in two markets (A and B). The firm's demand and cost conditions are described below.

$$\text{(Inverse Demand in Markets A and B) } p_A = 12 - 2q_A \text{ and } p_B = 6 - q_B$$

$$\text{(Total Cost) } TC = 2q, \text{ where } q = q_A + q_B.$$

- A. Derive the firm's optimal price and output levels in markets A and B.
- B. Show that the equilibrium price is higher in the market with the demand that is relatively more inelastic.

5. Assume a monopolist sells its output in a single market but production takes place in two plants (A and B). The firm's demand and cost conditions are described below.

$$\begin{aligned} \text{(Inverse Demand)} \quad p &= 12 - q \\ \text{(Total Cost for Plants A and B)} \quad TC_A &= (q_A^2)/2 \quad \text{and} \quad TC_B = q_B^2, \end{aligned}$$

where  $q = q_A + q_B$ .

- A. Derive the firm's optimal price and output levels for each plant.
- B. Show that the firm produces more output in the more efficient plant.

6. Assume a monopolist has two choice variables, price ( $p$ ) and quality ( $z$ ). The firm's demand and cost functions are described below:

$$\begin{aligned} \text{(Demand)} \quad q &= a - p + z \\ \text{(Total Cost)} \quad TC &= z^2, \end{aligned}$$

where parameter "a" is a positive constant.

- A. Suppose quality is fixed at 0. Determine the firm's profit maximizing price, output, and level of profits.
- B. Now suppose the firm can choose both  $p$  and  $z$ . Determine the firm's profit maximizing price, output, quality, and level of profits.
- C. Is the firm better off with control of product quality? Explain.

7. Consider a competitive market with the following demand and supply functions:

$$\begin{aligned} \text{(Demand)} \quad q_D &= D(p, m) \\ \text{(Supply)} \quad q_S &= S(p, t), \end{aligned}$$

where  $q_D$  is quantity demanded,  $q_S$  is quantity supplied,  $p$  is price,  $m$  is consumer income, and  $t$  is a per-unit tax. Assume positively sloped supply and negatively sloped demand functions.

- A. Use the implicit-function theorem to determine the impact of a change in  $m$  on the equilibrium price.
- B. Use the implicit-function theorem to determine the impact of a change in  $t$  on the equilibrium price.
- C. Under what conditions would it be **inappropriate** to use the implicit-function? Explain.

8. Assume a firm uses just one input ( $x$ ) to produce output and operates in competitive output and input markets. The firm's profit function is given below.

$$\pi = pq(x) - wx,$$

where  $\pi$  is profit and  $w$  is the price of the input.

- A. If the firm's goal is to choose its profit maximizing level of  $x$ , determine the firm's first and second order conditions for this problem. Provide an economic interpretation of these conditions.
- B. Use the implicit-function theorem to determine the impact of a change in  $w$  on the optimal  $x$ .

9. Consider a monopolist that faces the following demand and cost functions:

$$\begin{aligned} \text{(Demand)} \quad q &= q(p, \alpha) \\ \text{(Total Cost)} \quad C &= C(q, \beta), \end{aligned}$$

where  $q$  is output,  $p$  is price, and  $\alpha$  and  $\beta$  are parameters of the model.

- A. Use the implicit function theorem to compute the changes in the monopolist's optimal  $p$  for a marginal change in  $\alpha$  and in  $\beta$ .
- B. If  $\alpha$  is a policy variable that changes in a discrete way, making it impossible to use the implicit-function theorem, how would you obtain comparative static results? Explain and derive your result.

10. Consider the following normal-form games.

		Game 10A		Game 10B			
		Player 2		Player 2			
		L	R	X	Y	Z	
Player 1	U	1, 1	0, -3	A	8, 3	0, 4	4, 3
	D	-3, 0	0, 0	B	4, 2	1, 5	5, 3
				C	3, 7	0, 1	2, 0

- A. Identify all dominated strategies in games 10A and 10B.  
 B. Determine all dominant-strategy, iterated-dominant strategy, and strong and weak Nash equilibria in games 10A and 10B.

11. Assume Pat and Chris each own a soft-drink stand that is mobile and serve customers along a stretch of beach that is divided into 9 regions. Each region has the same area and an equal number of customers that are uniformly dispersed. Pat and Chris charge the same price for a soft drink. In a given period, sales from each region will be \$50, which would generate a total profit of \$10 per region. In a given period (i.e., a day), each vender can locate his or her stand in the center of any of the 1-9 regions, and customers walk to the nearest stand to purchase soft drinks. If customers in a given region are indifferent between the two stands (i.e., they are the same distance away), assume that half go to one stand and half go to the other. If each vender wants to maximize their own individual profit, determine their Nash equilibrium locations. Explain.

12. Consider the following normal-form game.

		Player 2		
		A	B	C
Player 1	A	0, 0	0, 5	0, 0
	B	5, 0	1, 0	-4, 0
	C	0, 0	0, -5	-5, -4

- A. Identify all iterated-dominant equilibria.  
 B. Identify all weak and strong Nash equilibria.

13. Consider a triopoly market, where firms 1, 2, and 3 have perfect and complete information and compete by simultaneously choosing output (as in Cournot). The demand and cost functions are defined as follows.

$$\text{(Inverse Demand) } p = 120 - (q_1 + q_2 + q_3)$$

$$\text{(Total Cost) } TC_1 = c_1 q_1, \quad TC_2 = TC_3 = 0,$$

where  $c_1$  is a positive constant and  $c_1 < 120$

- A. Determine the Nash equilibrium output, price and profit levels for each firm.  
 B. How does a change in  $c_1$  affect the equilibrium?

14. Consider a triopoly market, where firms 1, 2, and 3 have perfect and complete information and compete by simultaneously choosing prices (as in Bertrand). The demand and cost functions are defined as follows.

$$\begin{aligned} \text{(Firm } i \text{'s Demand)} \quad q_i &= 12 - p_i + d(p_j + p_k) \\ \text{(Total Cost)} \quad TC_i &= 0, \end{aligned}$$

where  $d$  is a parameter. Note that subscript  $i$  represents firms 1, 2, or 3 and subscripts  $j$  and  $k$  represent  $i$ 's other rivals.

- A. Determine the Nash equilibrium output, price and profit levels for each firm.
- B. What restrictions need to be placed on  $d$  to assure: (1) that firms participate in the game and (2) that the products are substitutes?
- C. How does a change in  $d$  affect the equilibrium?

15. Assume two firms (1 and 2) compete in marketing and have a choice of a low (L), medium (M), or high (H) expense marketing campaign. If their payoffs are described below, find all pure- and mixed-strategy Nash equilibria to the game.

		Firm 2		
		L	M	H
Firm 1	L	0, 0	-1, 1	0, 0
	M	1, -1	-2, -2	-3, -2
	H	0, 0	-2, -3	-3, -3

16. Briefly explain why the following equilibria are Nash equilibria: perfect competition, monopoly, Cournot duopoly.

17. Consider a Cournot duopoly (firms 1 and 2).

- A. Graph the best reply functions, with  $q_1$  on the vertical axis and  $q_2$  on the horizontal axis. Identify the Nash equilibrium.
- B. Prove that the Nash equilibrium in a Cournot duopoly will be stable when  $\pi_{11} \pi_{22} - \pi_{12} \pi_{21} > 0$ .

