Mth 442/542 Spring Term 2008

Homework 6. Due **Wednesday, May 21** in class.

A. 442 Students

p. 67 # 2.2.7

B. All Students

p. 67 # 2.2.8

p. 105 # 2.7.4 (You can use Maple's Groebner package.)

#4. *Defining term orders by weight matrices.* Let $M \in \mathcal{M}_{m,n}(\mathbb{Z})$ be such that $\ker I_M \cap \mathbb{Z}^n = \{0\}$, where $I_M : \mathbb{Q}^n \to \mathbb{Q}^m$ is given as usual by multiplication on the left by M.

For $\alpha = (\alpha(1), \ldots, \alpha(n)\,)$, let $\alpha \cdot M_i$ denote the usual dot product of the vector α with the ith *row* of M.

Define the relation $x_1^{\alpha(1)} \cdots x_n^{\alpha(n)} <_M \; x_1^{\beta(1)} \cdots x_n^{\beta(n)}$ if $\alpha \cdot M_1 < \beta \cdot M_1$, or if $\alpha \cdot M_i = \beta \cdot M_i$ for $1 \leq i < j$ and $\alpha \cdot M_j < \beta \cdot M_j$.

a.) Show that $<_M$ defines a term order on $k[x_1, \ldots, x_n]$. (See AL, p.23, 1.4.14 for related notions.)

b.) Identify the orders given by the following M:

$$(i.)\ M = I_n\,;\ (ii.)\ M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\ ;\ (iii.)\ M = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\ .$$

c.) The Maple Groebner package command to input an order determined by the matrix M with respect to variables (in order) x, y, z is `matrix(M, [x,y,z])`. Use `Basis` with a matrix order to find a Gröbner basis for $\langle r - x^4, u - x^3y, v - xy^3, w - y^4 \rangle$ with respect to an order which is: deglex on $y > x$; has both x, y greater than any of r, u, v, w; is degrevlex on $r > u > v > w$.

C. 542/649 only

p. 74, # 2.3.4